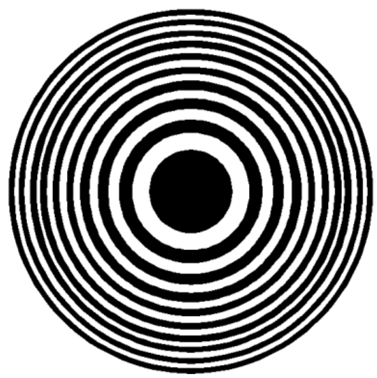
Answer on Question 50687, Physics, Optics

Question:

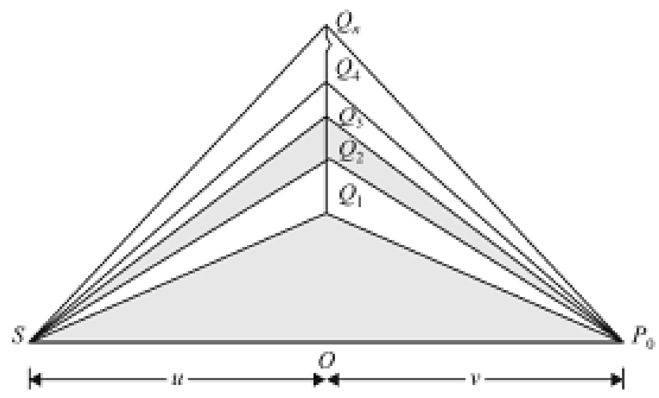
What is a zone plate? Show that it acts as a multi-foci converging lens.

Answer:

A zone plate is made by drawing concentric circles whose radii are proportional to the square root of natural numbers and alternate annular regions are shaded. The zone plate is used to focus light or for example to form images using X-rays and microwaves.



Let us show that zone plate acts as a multi-foci converging lens. In the picture below depicts the section of the zone plate normal to the plane of the paper. Suppose that S is a point source which emits monochromatic light waves and is at a distance u from the zone plate.



Let us study the effect of zone plate at the point P_0. The distance between P_0 and the zone plate is v. We suppose that the centre of the zone plate is at the point O and the radii of various zones are $OQ_1, OQ_2, OQ_3, \ldots, OQ_n$ such that each wave has to travel an additional distance of $\lambda/2$ from successive zones. Then we can write:

$$SQ_1 + Q_1P_0 = u + v + \frac{\lambda}{2}$$
$$SQ_2 + Q_2P_0 = u + v + \lambda$$
$$SQ_n + Q_nP_0 = u + v + \frac{n\lambda}{2} \quad (1)$$

By applying Pythagoras theorem to triangle SOQ_n we obtain:

$$SQ_n = \sqrt{SO^2 + OQ_n^2} = \sqrt{u^2 + r_n^2}$$

If radius of the n-th zone is much less than the distance of the source from the zone plate, we can use binomial expansion and neglect terms of order higher than $r_n^2/2u$:

$$SQ_n = u + \frac{r_n^2}{2u} + \cdots \quad (2)$$

Similarly, by applying Pythagoras theorem to triangle OQ_nP_0 we obtain:

$$Q_nP_0 = \sqrt{P_0O^2 + OQ_n^2} = \sqrt{v^2 + r_n^2}$$

If the distance of the point of observation from the zone plate is much greater than the radius of the n-th zone we obtain:

$$Q_nP_0 = v + \frac{r_n^2}{2v} + \cdots \quad (3)$$

Let us add expressions (2) and (3):

$$SQ_n + Q_nP_0 = u + \frac{r_n^2}{2u} + v + \frac{r_n^2}{2v} + \cdots$$

By combining this result with expression (1) we obtain:

$$u + \frac{r_n^2}{2u} + v + \frac{r_n^2}{2v} = u + v + \frac{n\lambda}{2}$$

After simplification we get:

$$r_n^2\left(\frac{1}{u} + \frac{1}{v}\right) = n\lambda$$

Let's identify term $f_n = \frac{r_n^2}{n\lambda}$ as a focal length of the zone plate, and we finally get the lens equation:

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f_n}$$

As follows from this equation, the zone plate acts like a multi-foci converging lens and forms a real image of S and P_0.