Question 5068

Two rocks are thrown off the edge of a cliff that is 15.0 m above the ground. The first rock is thrown upward, at a velocity of +12.0 m/s. The second is thrown downward, at a velocity of −12.0 m/s. Ignore air resistance. Determine (a) how long it takes the first rock to hit the ground and (b) at what velocity it hits. Determine (c) how long it takes the second rock to hit the ground and (d) at what velocity it hits.

Solution:

Lets begin with the **second stone**.
We are going to use this marking:

v - velocity, v_0 - the initial velocity, g - gravitational acceleration constant, S - displacement

We have:

$$S=15\mathrm{m}\,, v_0=12\,m/s\,, g=9.81\mathrm{m}/s^2 \quad (1)$$

The stone is being accelerated up by gravitation. Hence, for speed of the **second stone**:

$$v=v_0+g\cdot t \quad (2)$$

We obtain the displacement by integrating the speed by time:

$$S(t)=\int_0^t v\,dt=\int_0^t (v_0+g\cdot t)\,dt=v_0 t+\frac{g\cdot t^2}{2} \quad (3)$$

Using (3) and (1) , we get:

$$\frac{g\cdot t^2}{2}+12\mathrm{t}-15=0 \quad (4)$$

Here, t is the time, that takes the second stone to hit the ground. Solving the quadratic equation (4), we obtain two values for t:

$$t_1=0.9109, t_2=-3.3573 \quad (5)$$

Obviously, the time is a positive value, so , we take this value.

$$t=0.9109 \quad (6)$$

Now , we know how long does it take to the second stone to hit the ground. Lets find at what velocity it hits the ground. We have the change of velocity in time in (2). So, to obtain the latter velocity, we just put our time (6) into (2), and have:

$$v\approx 12\mathrm{m}/s+9.81\mathrm{m}/s^2\cdot 0.9109\mathrm{s}\approx 20.936\,m/s \quad (7)$$

We have solved the part of task about **second stone.** The answers are (6) and (7)

Lets move to the **first stone.** Its thrown upward, so for it we have such a situation: first, it moves upward for some time (lets say, t_1), and passes the displacement of S_1 . Then it stops (cause its accelerated down by g), and then it starts moving down , and passes the displacement of $L+S_1$, where L is the height of the cliff. Lets first find t_1 , S_1 .

For the first stone, while it moves upward we have:

$$v=v_0-g\cdot t \quad (1)$$

When it stops, v becomes zero. Using this we can find the time, during which the stone moves upward and stops. So, we equal v in (1) to zero, and obtain:

$$t_1=\frac{v_0}{g} \quad (2)$$

To find the law of motion of the **first stone**, we integrate (1) by time:

$$S(t)=\int_0^t v\,dt=\int_0^t (v_0-g\cdot t)\,dt=v_0 t-\frac{g\cdot t^2}{2} \quad (3)$$

Now, lets find S_1 . To do it, we put (2) into (3):

$$S_1=v_0 t-\frac{v_0^2}{2g} \quad (4)$$

The full time of movement of the first stone consists of t_1 , and the time (lets say, t_2), during which it passes the distance $L+S_1$. While moving down, the first stone has a v_0 of zero, and its accelerated up by g. So , the law of motion in that case is:

$$S(t)=\frac{g\cdot t^2}{2} \quad (5)$$

We use (5) to find t_2 :

$$L+S_1=L+v_0 t_1-\frac{v_0^2}{2g}=\frac{g\cdot t_2^2}{2}\Rightarrow t_2=\sqrt{\frac{2}{g}\cdot(L+\frac{v_0^2}{2g})} \quad (6)$$

The full time of movement t is then given by:

$$t=t_1+t_2=\frac{v_0}{g}+\sqrt{\frac{2}{g}\cdot(L+\frac{v_0^2}{2g})} \quad (7)$$

To find the speed at which the first rock hits the ground, we simply multiply g by t_2 :

$$v = g \cdot t_2 = \sqrt{2g \cdot \left(L + \frac{v_0^2}{2g}\right)} \quad (8)$$

We have an aswer to the task about first stone in (7) and (8). Putting numerical values there, we obtain:

$$t \approx 1.223 + 2.134 \approx 3.357\text{s}$$
$$v = 20.936\, m/s$$