

Answer on Question#50646, Physics, Molecular Physics | Thermodynamics

Derive an equation of state $pV^\gamma = \text{constant}$ for an adiabatic process and show that an adiabat is steeper than an isotherm.

Answer

Let us use the first law of thermodynamics $\delta Q = C_v dT + PdV$ in order to derive an equation of state for adiabatic process. For an adiabatic process $\delta Q = 0$, thus $C_v dT = -PdV$. Substituting the equation of ideal gas for one mole $P = \frac{RT}{V}$ into the right side of the previous equation, obtain

$$C_v dT = \frac{-RT}{V} dV, \text{ or } \frac{dT}{T} = \frac{-R}{C_v} \frac{dV}{V}. \text{ Integrating from both sides, obtain } \ln \frac{T_2}{T_1} = \frac{-R}{C_v} \ln \frac{V_2}{V_1}, \text{ or}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\frac{R}{C_v}} \text{ using the properties of logarithm. The last equation might be rewritten as}$$

$$TV^{\frac{R}{C_v}} = \text{const} \text{ or } TV^{\gamma-1} = \text{const}, \text{ where } \gamma = \frac{C_p}{C_v} \text{ (Here we also used } C_p = C_v + R \text{).}$$

Using $T = \frac{PV}{R}$ and last equation, obtain $PV^{\gamma-1+1} = PV^\gamma = \text{const}$.

The equation of isotherm is $PV = \text{const}$, thus if $\gamma = \frac{C_p}{C_v} > 1$, the adiabat $PV^\gamma = \text{const}$ is obviously