Answer on Question #50422, Physics, Mechanics | Kinematics | Dynamics

The minimum velocity at the lowest point, so that the string just slack at the highest point in a vertical circle of radius l.

(1)√gl(2)

(2)√3gl

(3)√5gl

(4)√7gl

Solution:

Consider a body of mass 'm' performs vertical circular motion about the centre and radius 'l'

As the motion is affected by the gravity the velocities of the body and tension in the string will be different at different points of the circle. Let v_1 be the velocity of the body at highest point. Let T be the tension at the highest points.

The forces acting on the body at the highest position are,

i) Weight of the body acting vertically downward direction,

ii) Tension T in the string, acting vertically downward direction.

Centripetal force acting on object at the highest position is provided partly by weight and partly by tension in the string.

$$T + mg = \frac{mv_1^2}{l}$$

There is certain velocity so called as critical velocity/minimum velocity (v) of object at highest point below which string become slack i.e. tension T vanishes (T=0).

$$mg = \frac{mv_1^2}{l}$$

$$v_1 = \sqrt{gl}$$

The decrease in potential energy between top -position and bottom position is

$$mgl - (-mgl) = 2\,mgl$$

This must be equal to the increase in kinetic energy, when particle move from highest point i.e.

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

Using law of conservation of energy,

$$2mgl = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$2mgl = \frac{1}{2}mv_2^2 - \frac{1}{2}mgl$$

$$4mgl = mv_2^2 - mgl$$

$$v_2^2 = 5gl$$

$$v_2 = \sqrt{5gl}$$

Equation gives required minimum velocity at lowest point.

Answer: $\sqrt{5gl}$.