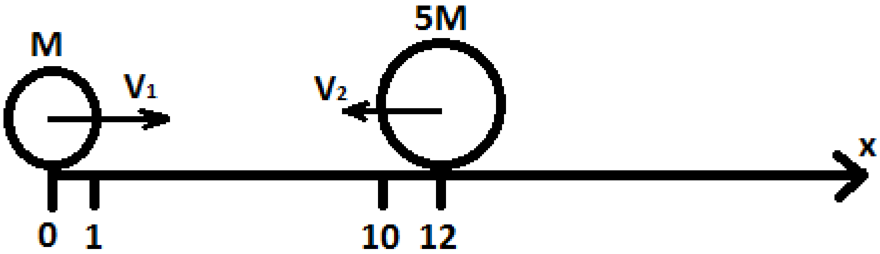
Answer on Question#50287 – Physics – Mechanics | Kinematics | Dynamics

(3) $7.5R$

Solution



Keep in mind that $x - axis$ actually passes through bodies' centers. Grading in units of R.

V_1 and V_2 denote velocities.

Due to Newton's 3rd law:

$$M\frac{d^2x_1}{dt^2} = -5M\frac{d^2x_2}{dt^2}$$

Thus,

$$\frac{d^2x_1}{dt^2} = -5\frac{d^2x_2}{dt^2}$$

, where x_1 and x_2 positions of the 1st and the 2nd bodies respectively.

Let us integrate both part with respect to time t.

$$\int \frac{d^2x_1}{dt^2}dt = -5\int \frac{d^2x_2}{dt^2}dt$$

$$\frac{dx_1}{dt} = V_1(t); \frac{dx_2}{dt} = V_2(t);$$

$$\int \frac{dV_1}{dt}dt = -5\int \frac{dV_2}{dt}dt$$

$$V_1 + C_1 = -5V_2 + C_2$$

, where C_1 and C_2 – constants of integration.

At initial moment of time $(t = 0)$ both velocities equal to zero, hence $C_1 = C_2 = 0$.

$$V_1 = -5V_2$$

Integrate with respect to time once more. Definite integral now. τ – time at which collision happens.

$$\int_0^\tau V_1 dt = -5\int_0^\tau V_2 dt$$

$$\int_0^\tau \frac{dx_1}{dt} dt = -5\int_0^\tau \frac{dx_2}{dt} dt$$

$$x_1(\tau) - x_1(0) = -5\big(x_2(\tau) - x_2(0)\big)$$

Recall, that due to our choice: $x_1(0) = 0;\ \ x_2(0) = 12;$

$$x_1(\tau) - 0 = -5(x_2(\tau) - 12)$$

$$x_1(\tau) = 60 - 5x_2(\tau)$$

Also, we know that minimal distance between spheres is nothing else, but the sum of its radii.

It means in our case: $x_2(\tau) - x_1(\tau) = 1 + 2 = 3.$

Thus, we can substitute $x_2(\tau) = 3 + x_1(\tau)$ into main equation above.

$$x_1(\tau) = 60 - 5\big(3 + x_1(\tau)\big)$$

$$x_1(\tau) = 60 - 15 - 5x_1(\tau)$$

$$6x_1(\tau) = 45$$

$$x_1(\tau) = 7.5$$

Recall that we get result in units of R.