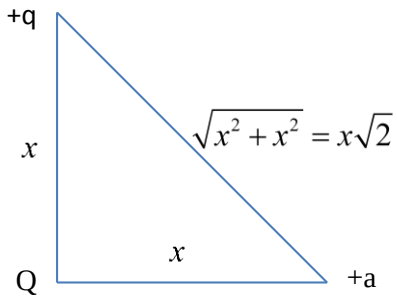
Answer on Question #50179 – Physics | Electrodynamics

three charges Q,+q,+a are placed at a vertices of a right angeles isoscale triangle.the net electrostatic energy of the configuration is zero if Q is equall to.....

Solution.

Electrostatic potential energy stored in a system of N charges $q_1, q_2, \dots, q_N$ at position $r_1, r_2, \dots, r_N$ respectively is $U = \sum_{i \neq j} k_e \frac{q_i q_j}{r_{ij}}$, where $k_e = \frac{1}{4\pi\varepsilon_0}$ is Coulomb's constant.

First case:

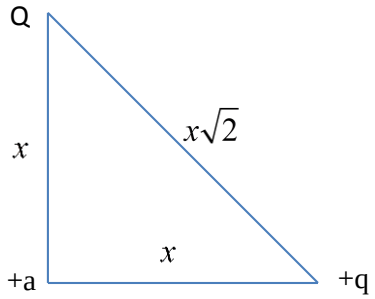


$$U = k_e\left(\frac{Qa}{x} + \frac{Qq}{x} + \frac{qa}{\sqrt{2}x}\right) = \frac{k_e}{x}\left(Q(a+q) + \frac{qa}{\sqrt{2}}\right) = 0$$

$$Q(a+q) + \frac{qa}{\sqrt{2}} = 0$$

$$Q = \frac{-qa}{\sqrt{2}(a+q)}$$

Second case:

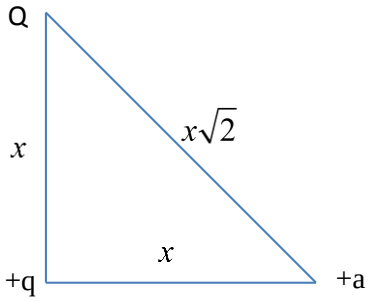


$$U = k_e\left(\frac{Qa}{x} + \frac{aq}{x} + \frac{qQ}{\sqrt{2}x}\right) = \frac{k_e}{\sqrt{2}x}\left(Q\left(\sqrt{2}a+q\right) + \sqrt{2}qa\right) = 0$$

$$Q\left(\sqrt{2}a+q\right) + \sqrt{2}qa = 0$$

$$Q = \frac{-qa}{a + \frac{q}{\sqrt{2}}}$$

Third case:



$$U = k_e\left(\frac{Qq}{x} + \frac{aq}{x} + \frac{aQ}{\sqrt{2}x}\right) = \frac{k_e}{\sqrt{2}x}\left(Q\left(\sqrt{2}q+a\right) + \sqrt{2}qa\right) = 0$$

$$Q\left(\sqrt{2}q+a\right) + \sqrt{2}qa = 0$$

$$Q = \frac{-qa}{q + \frac{a}{\sqrt{2}}}$$

Answer:

First case: $Q = \frac{-qa}{\sqrt{2}(a+q)}$; second case: $Q = \frac{-qa}{a+\frac{q}{\sqrt{2}}}$; third case: $Q = \frac{-qa}{q+\frac{a}{\sqrt{2}}}$.