Answer on Question 50057, Physics, Mechanics | Kinematics | Dynamics

Question:

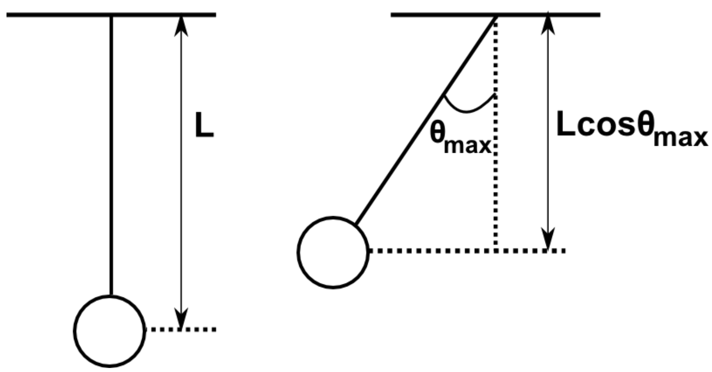
A simple pendulum with a length of 2.23m and a mass of 6.74kg is given an initial speed of 2.06m/s at its equilibrium position. Assume it undergoes simple harmonic motion, and determine its:

(a) period,

(b) total energy,

(c) maximum angular displacement.

Solution:



a) From the definition of the Huygens law for the period of the pendulum we have:

$$T = 2\pi\sqrt{\frac{L}{g}},$$

where T is the period, L is the length of the pendulum and g is the gravitational acceleration. So, the period of the pendulum will be:

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{2.23m}{9.8\frac{m}{s^2}}} = 3s.$$

b) We can find the total energy of the pendulum from the law of the conservation of energy. At the equilibrium position the potential energy of the pendulum is zero, so the total energy of the pendulum is its kinetic energy:

$$E_{total} = KE = \frac{1}{2}mv^2 = 0.5 \cdot 6.74kg \cdot \left(2.06\frac{m}{s}\right)^2 = 14.3J.$$

c) In order to find the maximum angular displacement we again use the law of the conservation of energy. The pendulum will rise up until the heigth where its velocity is zero. At this heigth all its kinetic energy has become potential energy:

$$E_{total} = PE = mgh = mg\left(L - L\cos\theta_{\max}\right) = mgL\left(1 - \cos\theta_{\max}\right).$$

Hence, from this formula we can find the maximum angular displacement:

$$\cos\theta_{\max} = 1 - \frac{E_{total}}{mgL} = 1 - \frac{14.3J}{6.74kg \cdot 9.8\frac{m}{s^2} \cdot 2.23m} = 0.903.$$

$$\theta_{\max} = \arccos\left(0.903\right) = 25.4^\circ = 25.4^\circ\frac{\pi}{180^\circ} = 0.443rad.$$

Answer:

a) $T = 3s.$

b) $E_{total} = 14.3J.$

c) $\theta_{\max} = 0.443rad.$