Answer on Question 49943, Physics, Mechanics | Kinematics | Dynamics

Question:

A projectile is short from the edge of a cliff 125m above ground level with an initial speed of 105 m/s at an angle of 37° with the horizontal.
1) Determine the time taken by the projectile to hit point P at ground level.
2) Determine the range x of the projectile as measured from the base of the cliff.
3) At the instant just before the projectile hits point P, find the horizontal and vertical components of the velocity.
4)The magnitude of the velocity.
5) The angle made by the velocity vector with the horizontal.

Solution:

1) Let's write the projections of the initial speed of the projectile on axis x and y:

$$v_{0x} = v_0 \cos 37^\circ = 105\frac{m}{s} \cdot \cos 37^\circ = 83.85\frac{m}{s},$$
$$v_{0y} = v_0 \sin 37^\circ = 105\frac{m}{s} \cdot \sin 37^\circ = 63.2\frac{m}{s}.$$

Let's obtain the time taken by the projectile to hit point P at ground level:

$$y = v_{0y}t + \frac{1}{2}gt^2,$$

$$4.9t^2 + 63.2t - 125 = 0,$$

This equation has two roots:

$$t_1 = \frac{-63.2 - \sqrt{6444.24}}{2 \cdot 4.9} = -14.64,$$
$$t_2 = \frac{-63.2 + \sqrt{6444.24}}{2 \cdot 4.9} = 1.74.$$

Because time can't be negative the correct answer is $t = 1.74s.$

2) As we known t we can calculate the range x of the projectile as measured from the base of the cliff:

$$x = v_{0x}t = 83.85\frac{m}{s}\cdot 1.74s = 146m$$

3) Now, we find the vertical component of velocity just before the projectile hits point P (we assume that axis y directed upward from the cliff):

$$v_y = v_{0y} + gt = 63.2\frac{m}{s} + 9.8\frac{m}{s^2}\cdot 1.74s = 80.25\frac{m}{s}.$$

The horizontal and vertical components of the velocity are: $v_x = 83.85\frac{m}{s}, v_y = 80.25\frac{m}{s}.$

4) So, when the projectile hits the ground the velocity components are $v_x = 83.85\frac{m}{s}, v_y = 80.25\frac{m}{s}.$ Hence, we can find the magnitude of the velocity:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(83.85\frac{m}{s}\right)^2 + \left(80.25\frac{m}{s}\right)^2} = 116\frac{m}{s}.$$

5) Finally, we can find the angle made by the velocity vector with the horizontal:

$$\tan\alpha = \frac{v_y}{v_x} = \frac{80.3\frac{m}{s}}{83.85\frac{m}{s}} = 0.9576.$$

$$\alpha = \arctan\left(0.9576\right) = 43.7^\circ.$$

Answer:

1) $t = 1.74s.$

2) $x = 146m.$

3) $v_x = 83.85\frac{m}{s}, v_y = 80.25\frac{m}{s}.$

4) $v = 116\frac{m}{s}.$

5) $\alpha = 43.7^\circ.$