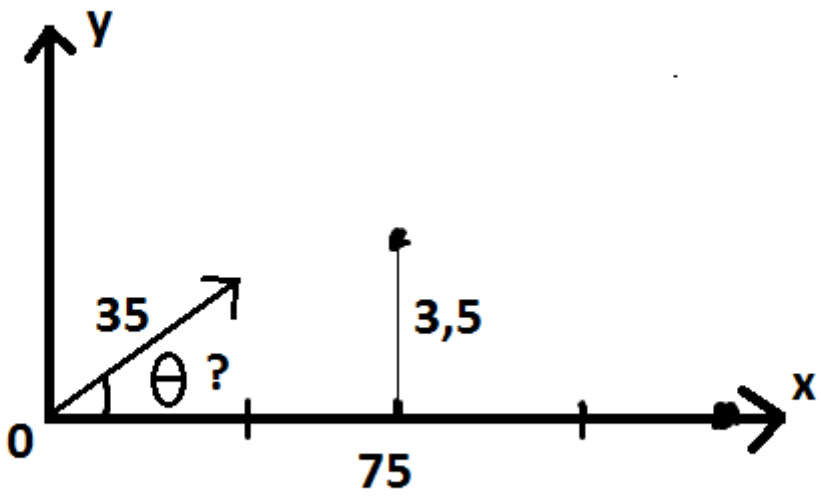
Answer on Question#49931 – Physics – Mechanics | Kinematics | Dynamics

$$\theta = \frac{1}{2} asin\left(\frac{75g}{V^2}\right) \approx 18.88°$$

Stone will go up of the buckle.

Solution



Equations of motion:

$$m\frac{d^2x}{dt} = 0$$

$$m\frac{d^2y}{dt^2} = -mg$$

Initial conditions:

$$x = 0; \quad V_x(0) = V\cos(\theta); \quad y(0) = 0; \quad V_y(0) = V\sin(\theta);$$

Explicit solutions of the equations above:

$$x = C_1 + C_2 t$$

$$y = -\frac{gt^2}{2} + C_3 t + C_4$$

Use initial conditions:

$$x(0) = C_1 + C_2(0) = C_1 = 0$$

$$V_x(0) = \frac{dx}{dt}(0) = C_2 = V\cos(\theta)$$

$$y(0) = -\frac{g(0)^2}{2} + C_3(0) + C_4 = C_4 = 0$$

$$V_y(0) = \frac{dy}{dt}(0) = -g(0) + C_3 = C_3 = V\sin(\theta)$$

Thus, we have:

$$x(t) = V\cos(\theta)\,t$$

$$y(t) = V\sin(\theta)\,t - \frac{gt^2}{2}$$

We know that at some time t_1:

$$x(t_1) = V\cos(\theta)\,t_1 = 75$$

$$y(t_1) = V\sin(\theta)\,t_1 - \frac{gt_1^2}{2} = 0$$

Solve last equation for t_1:

$$V\sin(\theta)\,t_1 - \frac{gt_1^2}{2} = 0$$

$$t_1\left(V\sin(\theta) - \frac{gt_1}{2}\right) = 0$$

$$t_{11} = 0$$

$$t_{12} = \frac{2V\sin(\theta)}{g}$$

Obviously, we need second solution to use. Put it into equation $x(t_1)$ to determine angle θ:

$$V\cos(\theta)t_1 = 75$$

$$\frac{V\cos(\theta)\,2Vsin(\theta)}{g} = 75$$

$$2\sin(\theta)\cos(\theta) = \sin(2\theta)$$

$$\frac{V^2}{g}\sin(2\theta) = 75$$

$$2\theta = \operatorname{asin}\left(\frac{75g}{V^2}\right)$$

$$\theta = \frac{1}{2}\operatorname{asin}\left(\frac{75g}{V^2}\right)$$

Substitute

$$V = 35; \quad g = 10;$$

$$\theta = \frac{1}{2}\operatorname{asin}\left(\frac{75 * 10}{35^2}\right) \approx 18.88°$$

As soon as velocity along x – constant, stone pass middle point at time $\frac{t_1}{2}$. Calculate $y\left(\frac{t_1}{2}\right)$:

$$y\left(\frac{t_1}{2}\right) = V\sin(\theta)\frac{t_1}{2} - \frac{g}{8}t_1^2 = \frac{\left(V\sin(\theta)\,t_1 - \frac{g}{2}t_1^2\right)}{2} + \frac{g}{8}t_1^2$$

Recall that

$$V\sin(\theta)\,t_1 - \frac{g}{2}t_1^2 = 0$$

Then

$$y\left(\frac{t_1}{2}\right) = \frac{g}{8}t_1^2 = \frac{g}{8}\left(\frac{2V\sin(\theta)}{g}\right)^2 = \frac{g}{8}\frac{4V^2}{g^2}\sin^2\left(\frac{1}{2}\operatorname{asin}\left(\frac{75g}{V^2}\right)\right) = \frac{V^2}{2g}\sin^2\left(\frac{1}{2}\operatorname{asin}\left(\frac{75g}{V^2}\right)\right)$$

Substitute

$$V = 35; \quad g = 5;$$

$$y(t_1) \approx 6.41 > 3$$