Answer on Question #49916-Physics-Mechanics-Kinematics-Dynamics

A particle is projected from ground at an angle θ with horizontal with speed u. The ratio of radius of curvature of its trajectory at point of projection to radius of curvature at maximum height is? Options are

A)1/sin^2θ cosθ B)cos^2θ C)1/sin^3θ D)1/cos^3θ

Solution

$x(t) = u\cos\theta\, t, y(t) = u\sin\theta\, t - \frac{1}{2}gt^2.$

Radius of curvature is

$$R = \left|\frac{\left[\left(x'(t)\right)^2 + \left(y'(t)\right)^2\right]^{3/2}}{x'(t)y''(t) - y'(t)x''(t)}\right|.$$

$x'(t) = u\cos\theta\,, x''(t) = 0.$

$y'(t) = u\sin\theta - gt, y''(t) = -g.$

At $t = 0$

$x'(0) = u\cos\theta\,, x''(0) = 0, y'(0) = u\sin\theta\,, y''(0) = -g.$

At $t = \frac{u\sin\theta}{g}$

$x' = u\cos\theta\,, x'' = 0, y' = 0, y'' = -g.$

Substituting values we get

$$R(t = 0) = \frac{u^2}{g\cos\theta}.$$

$$R\left(t = \frac{u\sin\theta}{g}\right) = \frac{u^2\cos^2\theta}{g}.$$

The ratio is

$$\frac{R(t = 0)}{R\left(t = \frac{u\sin\theta}{g}\right)} = \frac{1}{\cos^3\theta}.$$

Answer: D) $\frac{1}{\cos^3\theta}$.