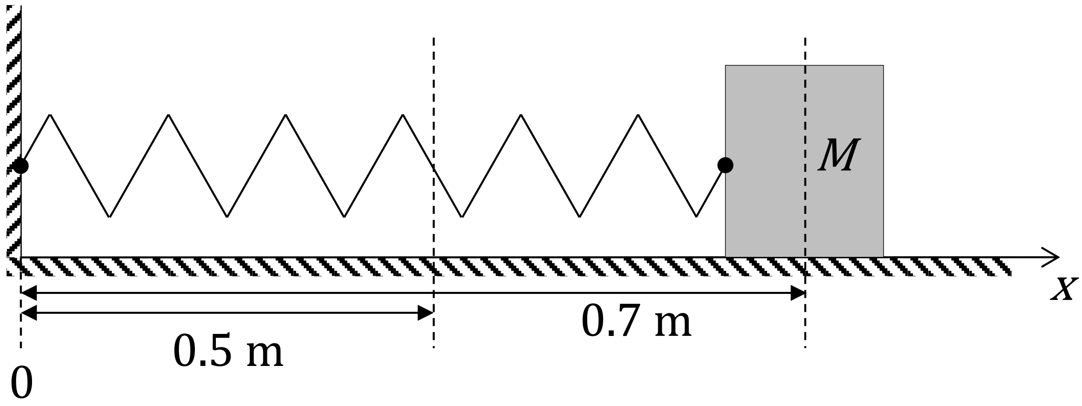
Answer on Question#49746 - Physics - Mechanics - Kinematics - Dynamics

A spring with a mass of 4 kg has natural length 0.5 m. A force of 25.6 N is required to maintain it stretched to a length of 0.7 m. If the spring is stretched to a length of 0.7 m and then released with initial velocity 0, find the position of the mass at any time.

Solution:



Since the force of 25.6 N is required to maintain the spring stretched by the length of 0.2 m, the stiffness of the string has the following value

$$k = \frac{25.6\ \mathrm{N}}{0.2\ \mathrm{m}} = 128\ \frac{\mathrm{N}}{\mathrm{m}}$$

According to the 2 Newton's law the equation of motion of the mass (under the restoring force) can be written as follows

$$M\ddot{x} = -k(x - 0{,}5\ \mathrm{m})$$

where $M = 4$ kg and x is the position of the mass. The solution of this equation is

$$x = 0{,}5\ \mathrm{m} + A\sin\frac{k}{M}t + B\cos\frac{k}{M}t \qquad (1)$$

where A and B are some constants. We can determine them from the initial conditions, which are

$$x(0) = 0.7\ \mathrm{m},\ \ \dot{x}(0) = 0\frac{\mathrm{m}}{\mathrm{s}}$$

Using the first condition we obtain

$$0{,}7\ \mathrm{m} = 0{,}5\ \mathrm{m} + B$$

It gives us the value of B:

$$B = 0{,}2\ \mathrm{m}$$

Taking the derivative of (1) we obtain

$$\dot{x} = A\frac{k}{M}\cos\frac{k}{M}t - B\frac{k}{M}\sin\frac{k}{M}t$$

Using second condition we obtain

$$\dot{x}(0) = A\frac{k}{M} = 0$$

It gives us the value of A:

$$A = 0$$

Therefore, the position of the mass at any time is given by

$$x = 0{,}5\text{m} + 0{,}2\text{m}\cdot\cos\frac{128\frac{\text{N}}{\text{m}}}{4\text{kg}}t = 0{,}5\text{m} + 0{,}2\text{m}\cdot\cos(32\text{s}^{-1}\cdot t)$$

Answer: $x = 0{,}5\text{m} + 0{,}2\text{m}\cdot\cos(32\text{s}^{-1}\cdot t)$.