Answer on Question #49675, Physics, Mechanics - Kinematics - Dynamics

A tennis player wishes to return the ball so it just passes over the top of the net at a point where the net is 1.0 m high. He can return the ball at a speed of 14 ms-1 from a position that is 0.4 m vertically below the top of the net and is 8.0 m from the point where he intends the ball to cross the net.
a. At what angles to the horizontal could the player strike the ball in order to do this?

Solution

Equation OX axis:

$$x = v_{0x}t => t = \frac{x}{v_{0x}}$$

Equation OY axis:

$$y = v_{0y}\frac{x}{v_{0x}} - \frac{g\left(\frac{x}{v_{0x}}\right)^2}{2} = \frac{\sin\alpha}{\cos\alpha}x - \frac{gx^2}{2v_{0x}^2}$$

We need to solve this equation and find α

$$y = \tan\alpha\, x - \frac{gx^2}{2v_0^2\cos^2\alpha} = \tan\alpha\, x - \frac{gx^2(1+\tan^2\alpha)}{2v_0^2}$$

$$\frac{gx^2}{2v_0^2}\tan^2\alpha - x\tan\alpha + \left(y + \frac{gx^2}{2v_0^2}\right) = 0$$

This is a quadratic equation with solution:

$$\tan\alpha = \frac{x \pm \sqrt{x^2 - 4\frac{gx^2}{2v_0^2}\left(y + \frac{gx^2}{2v_0^2}\right)}}{2\frac{gx^2}{2v_0^2}} = \frac{v_0^2 \pm \sqrt{v_0^4 - 2v_0^2 g\left(y + \frac{gx^2}{2v_0^2}\right)}}{gx} = \frac{v_0^2 \pm \sqrt{v_0^4 - g(2yv_0^2 + gx^2)}}{gx}$$

$$\tan\alpha = \frac{\left(14\frac{m}{s}\right)^2 \pm \sqrt{\left(14\frac{m}{s}\right)^4 - 9.8\frac{m}{s^2}\cdot\left(2\cdot(1.0m - 0.4m)\cdot\left(14\frac{m}{s}\right)^2 + 9.8\frac{m}{s^2}\cdot(8.0m)^2\right)}}{9.8\frac{m}{s^2}\cdot 8.0m}$$
$$\approx 4.7; 0.29$$

Then the range of angles is:

$$\alpha = \operatorname{atan} 4.7\,;\ \alpha = \operatorname{atan} 0.29$$

$$\alpha = 78°; \alpha = 16°$$

Answer: $\boldsymbol{\alpha = 78°; 16°}$