Answer on Question#49674 - Physics - Mechanics - Kinematics - Dynamics

A tennis player wishes to return the ball so it just passes over the top of the net at a point where the net is $H = 1.0\text{m}$ high. He can return the ball at a speed of $v_0 = 14\frac{\text{m}}{\text{s}}$ from a position that is $\Delta h = 0.4\text{m}$ vertically below the top of the net and is $l_0 = 8.0\text{m}$ from the point where he intends the ball to cross the net.

a. At what angles to the horizontal could the player strike the ball in order to do this?

b. In each case what will be the total horizontal distance travelled by the ball before it strikes the ground on the other side of the net?

c. Without further calculation, sketch the trajectories in each case.

<u>Solution:</u>

The initial position is $h_0 = H - \Delta h = 0.6\text{m}$ high. Assuming that the ball was struck at angle φ to the horizontal, dependences of the height h and horizontal displacement l of the ball on time are

$$\begin{cases} l = v_0 \cos\varphi \cdot t \\ h = h_0 + v_0 \sin\varphi \cdot t - \dfrac{g \cdot t^2}{2} \end{cases} \qquad (1)$$

When the ball passes over the net these equations give us the following

$$\begin{cases} l_0 = v_0 \cos\varphi \cdot t \\ H = h_0 + v_0 \sin\varphi \cdot t - \dfrac{g \cdot t^2}{2} \end{cases}$$

Expressing t from the first equation and substituting it into the second we obtain

$$\begin{cases} t = \dfrac{l_0}{v_0 \cos\varphi} \\ H = h_0 + v_0 \sin\varphi \dfrac{l_0}{v_0 \cos\varphi} - \dfrac{g \cdot l_0^2}{2(v_0 \cos\varphi)^2} \end{cases}$$

It's known that

$$\frac{1}{\cos^2\varphi} = 1 + \tan^2\varphi$$

So the second equation can be rewritten in the following way

$$H = h_0 + l_0 \tan\varphi - \frac{g \cdot l_0^2}{2v_0^2}(1 + \tan^2\varphi)$$

or

$$\tan^2\varphi - \frac{2v_0^2}{g \cdot l_0}\tan\varphi + \frac{2v_0^2 \Delta h}{g \cdot l_0^2} + 1 = 0$$

Substituting all given parameters and assuming that $g = 10\,\frac{\mathrm{m}}{\mathrm{s}^2}$ we obtain

$$\tan^2 \varphi - 4.9 \tan \varphi + 1.245 = 0$$

This equation has two roots

$$\tan \varphi = \left[\begin{matrix} 4.63 \\ 0.27 \end{matrix}\right.$$

Since we are only interested in angles which lie in the interval $\left[0, \frac{\pi}{2}\right]$, we have only two possible angles

$$\varphi_1 = \operatorname{atan} 4.63 = 77.8°$$
$$\varphi_2 = \operatorname{atan} 0.27 = 15.0°$$

Let's first consider the case when $\varphi_1 = 77.8°$.

To find the total horizontal distance travelled by the ball before it strikes the ground we should find the time it took the ball to reach the ground ($h = 0$). To do this we have to solve the second equation of the system (1) with respect to time t.

$$h_0 + v_0 \sin \varphi_1 \cdot t - \frac{g \cdot t^2}{2} = 0$$

or

$$t^2 - \frac{2v_0 \sin \varphi_1}{g} t - \frac{2h_0}{g} = 0$$

Substituting given parameters and $\varphi_1 = 77.8°$ we obtain

$$t^2 - 2.74t - 0.12 = 0$$

This equation has only one physical root (not negative)

$$t = 2.78\mathrm{s}$$

Using the first equation of the system (1) we obtain the total horizontal distance

$$L_1 = v_0 \cos \varphi_1 \cdot t = 8.3\mathrm{m}$$

Let's now consider the case when $\varphi_2 = 15.0°$.

We have to solve a similar equation

$$t^2 - \frac{2v_0 \sin \varphi_2}{g} t - \frac{2h_0}{g} = 0$$

Substituting given parameters and $\varphi_1 = 15.0°$ we obtain

$$t^2 - 0.72t - 0.12 = 0$$

This equation has only one physical root (not negative)

$$t = 0.86\text{s}$$

Using the first equation of the system (1) we obtain the total horizontal distance

$$L_2 = v_0 \cos\varphi_2 \cdot t = 11.6\text{m}$$

Answer:

a. $\varphi_1 = 77.8°$
   $\varphi_2 = 15.0°$
b. $L_1 = 8.3\text{m}$
   $L_2 = 11.6\text{m}$
c.