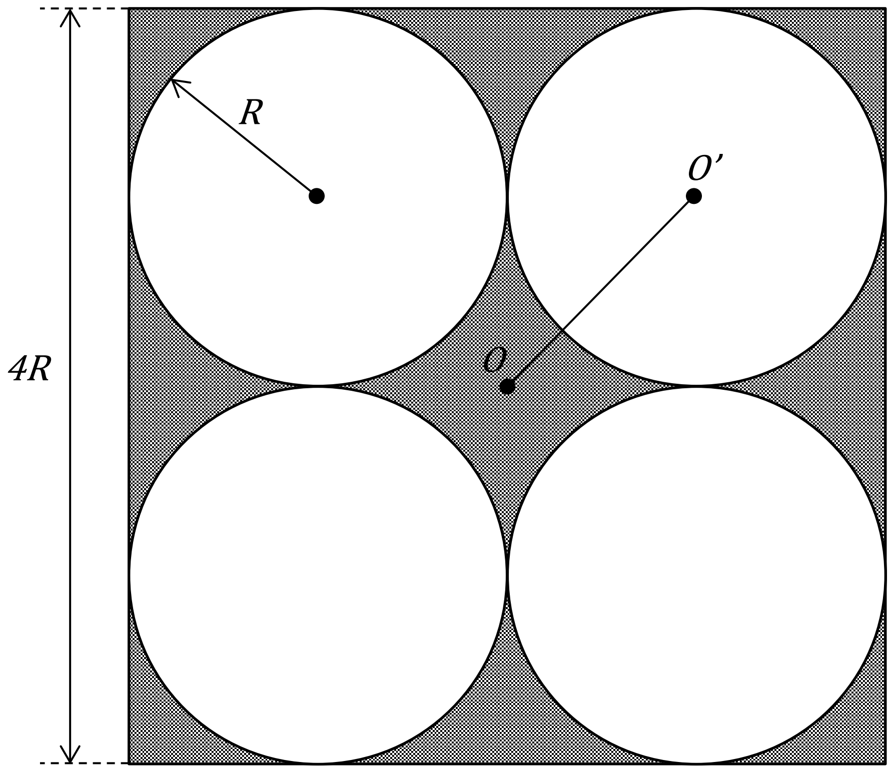
Answer on Question#49436 - Physics - Mechanics - Kinematics - Dynamics

4 holes of radius R are cut from thin square plate of side $4R$ and mass m. The moment of inertia of remaining part about z axis is?

Solution:



The moment of inertia of the thin square of side l and mass M (the axis of rotation passes through its center) is given by

$$I_s = \frac{Ml^2}{6} \quad (1)$$

The momentum of inertia of the disk of radius r and mass M (the axis of rotation passes through its center) is given by

$$I_d = \frac{Mr^2}{2} \quad (2)$$

The mass of the disk produced by holing the square plate has the following value

$$m_d = \frac{\pi R^2}{(4R)^2} m$$

where m is the mass of the square plate, $(4R)^2$ is the area of the plate, πR^2 is the area of the disk.

To determine the momentum of inertia of the remaining part which is shown in the figure above we'll first determine the momentum of inertia of the disk with the axis of rotation passing through the point O. To this end we'll apply the Huygens-Steiner theorem. According to this theorem the momentum of inertia of such disk is given by

$$I_d^O = I_d^{O'} + m_d(OO')^2$$

where $I_d^{O'}$ is given by the formula (2) with $r = R$ and $M = m_d$. Since the distance OO' has the value of $\sqrt{2}R$, the momentum of inertia of the disk is given by

$$I_d^O = \frac{m_d R^2}{2} + m_d\left(\sqrt{2}R\right)^2 = \frac{5}{2} m_d R^2 = \frac{5}{2}\frac{\pi R^2}{(4R)^2} mR^2 = \frac{5\pi}{32} mR^2$$

The momentum of inertia of four such disks (4 holes) has 4 times larger value than I_d^O and is given by

$$I_4 = 4 I_d^O = \frac{5\pi}{8} mR^2$$

The momentum of inertia of the square plate is given by the formula (1) with $M = m$ and $l = 4R$:

$$I_p = \frac{m(4R)^2}{6} = \frac{8}{3} mR^2$$

The momentum of inertia of the remaining part is obtained by subtracting I_4 from I_p:

$$I = I_p - I_4 = \frac{8}{3} mR^2 - \frac{5\pi}{8} mR^2 = \frac{64 - 15\pi}{24} mR^2$$

Answer: $\frac{64-15\pi}{24} mR^2$.