Answer on Question #49236-Physics-Mechanics-Kinematics-Dynamics

The amount of work done in pumping the water out of a cubical vessel of height $h = 1m$ is nearly (Taking $g = 10\frac{m}{s^2}$)

(1)5000

(2)10000

(3)5

(4)10

All options are in Joules.

Solution

Pumping liquid out of the top of a tank requires work because the liquid is moving against gravity. To calculate this, we imagine the work required to lift small disks of liquid up and out of the tank. So we are lifting a series of masses against gravity and allowing the liquid to spill out once the top is reached. We are asked to calculate the work performed in all of this activity.

Recall that weight = mass times the force of gravity ($W = mg$), where W is weight, m is mass, and g is the gravitational constant. What varies in these systems is the distance each disc needs to be lifted, measured by taking the total height, H, and subtracting from this the present height of the remaining liquid, x, and the volume of each disc.

The density of water is $\rho = 1000\frac{kg}{m^3}$.

For a cubical vessel

$$F = \rho g h^2 dx.$$

Since Work = ($F \cdot distance$) and the distance is $h - x$, then

$$W = \int_0^h \rho g h^2 (h - x)dx = \rho g h^2 \int_0^h (h - x)dx = \rho g h^2 \left(hx - \frac{x^2}{2}\right)_0^h = \rho g h^2 \left(h^2 - \frac{h^2}{2}\right) = \frac{1}{2}\rho g h^3.$$

$$W = \frac{1}{2} \cdot 1000\frac{kg}{m^3} \cdot 10\frac{m}{s^2} \cdot (1m)^3 = 5000\,J.$$

Answer: (1) 5000J.