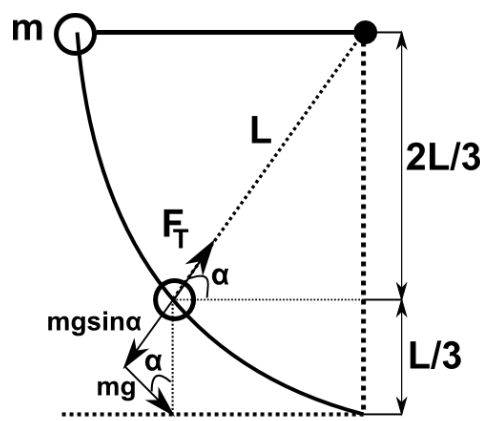
Answer on Question 49101, Physics, Mechanics | Kinematics | Dynamics

Question:

4. A steel ball of mass m is fastened to a light cord of length L and released when the cord is horizontal. At the bottom of its path, the ball strikes a hard plastic block of mass M = 3m, initially at rest on a frictionless surface. The collision is elastic.
(a) Find the tension in the cord when the ball's height above its lowest position is L/3. Write your answer in terms of m and g.
(b) Find the speed of the block immediately after the collision.
(c) To what height h will the ball rebound after the collision?

Solution:

a) Let's draw a free-body diagram:



So, let's write the forces acting on the ball:

$$F_T - mg\sin\alpha = F_{centripetal} = \frac{mv^2}{L}.$$

We can see that we have two unknowns in this equation mv^2 and $\sin\alpha$.

Let's write the law of conservation of energy to find mv^2. Then we have:

$$KE_i + PE_i = KE_f + PE_f,$$

$$0 + mgL = \frac{1}{2}mv^2 + mg\frac{L}{3},$$

$$mv^2 = \frac{4}{3}mgL.$$

From the triangle in the free-body diagram we can see that $\sin\alpha = \frac{2}{3}L \Big/ L = \frac{2}{3}.$

So, after substituting mv^2 and $\sin\alpha$ into first equation we obtain:

$$F_T = \frac{2}{3}mg + \frac{4}{3}mg = 2mg.$$

b) First we find the velocity of the ball before it hits the block. Let's write the law of conservation of energy:

$$KE_i + PE_i = KE_f + PE_f,$$

$$0 + mgL = \frac{1}{2}mv^2 + 0,$$

$$v = \sqrt{2gL}.$$

So, because we have elastic head-on collision and kinetic energy is conserved we can obtain the velocity of the block after collision:

$$v_2' = \frac{2m_1}{m_1 + M}v = \frac{2m_1}{m_1 + 3m_1}\sqrt{2gL} = \frac{2m_1}{4m_1}\sqrt{2gL} = \frac{1}{2}\sqrt{2gL}.$$

c) We can find h from the law of conservation of energy:

$$KE_i + PE_i = KE_f + PE_f,$$

$$\frac{1}{2}mv_1'^2 + 0 = 0 + mgh,$$

$$h = \frac{v_1'^2}{2g}.$$

We can find the velocity of the ball after collision from the formula:

$$v_1' = \frac{m_1 - M}{m_1 + M}v_1 = \frac{m_1 - 3m_1}{m_1 + 3m_1}\sqrt{2gL} = -\frac{2m_1}{4m_1}\sqrt{2gL} = -\frac{1}{2}\sqrt{2gL}.$$

After substituting v_1' into the formula for h we obtain:

$$h = \frac{\left(-\frac{1}{2}\sqrt{2gL}\right)^2}{2g} = \frac{\frac{1}{4}2gL}{2g} = \frac{1}{4}L.$$

Answer:

a) $F_T = 2mg.$

b) $v_2' = \frac{1}{2}\sqrt{2gL}.$

c) $h = \frac{1}{4}L.$

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