Answer on Question #49042-Physics-Other

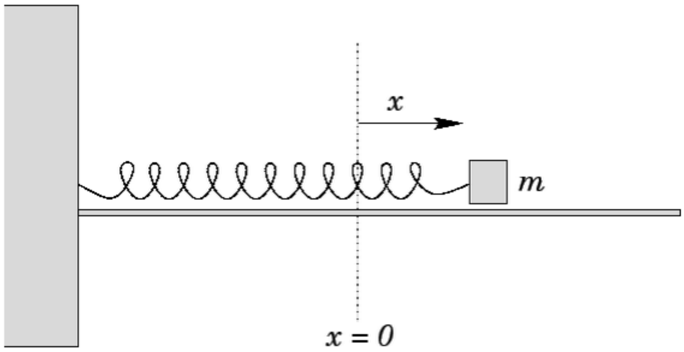
We know generally that the direction of displacement and acceleration is same. But for periodic motion, in case of a simple pendulum, the acceleration is proportional and opposite directed to displacement. What is the reason please explain detail?

The equation is "a is proportional to $-x$" where a is acceleration and x is displacement.

Answer

We consider a simple harmonic motion with an object attached to a spring. (In case of a simple pendulum the angle θ subtended between the string and the downward vertical is a coordinate in equation of motion.)

Consider a mass m which slides over a horizontal frictionless surface. Suppose that the mass is attached to a light horizontal spring whose other end is anchored to an immovable object. See figure.



Mass on a spring

Let x be the extension of the spring: i.e., the difference between the spring's actual length and its unstretched length. Obviously, x can also be used as a coordinate to determine the horizontal displacement of the mass.

The equilibrium state of the system corresponds to the situation where the mass is at rest, and the spring is unextended (i.e., $x = 0$). In this state, zero net force acts on the mass, so there is no reason for it to start to move. If the system is perturbed from this equilibrium state (i.e., if the mass is moved, so that the spring becomes extended) then the mass experiences a restoring force given by Hooke's law:

$$F = -kx.$$

Here, $k > 0$ is the force constant of the spring. The negative sign indicates that F is indeed a restoring force. Note that the magnitude of the restoring force is directly proportional to the displacement of the system from equilibrium (i.e., $f \propto x$). Of course, Hooke's law only holds for small spring extensions. Hence, the displacement from equilibrium cannot be made too large. The motion of this system is representative of the motion of a wide range of systems when they are slightly disturbed from a stable equilibrium state.

Newton's second law gives following equation of motion for the system:

$$ma = -kx \rightarrow a = -\frac{k}{m}x.$$

So, the acceleration is proportional and opposite directed to displacement.