Answer on Question #48972, Physics, Optics

A laser whose wavelength is $\lambda_a = 4.7 \times 10^{-7}$ m shines through a diffraction grating. The light hits a screen L = 3 m away and fourth order maximum is detected 0.52 m on either side of the central maximum.

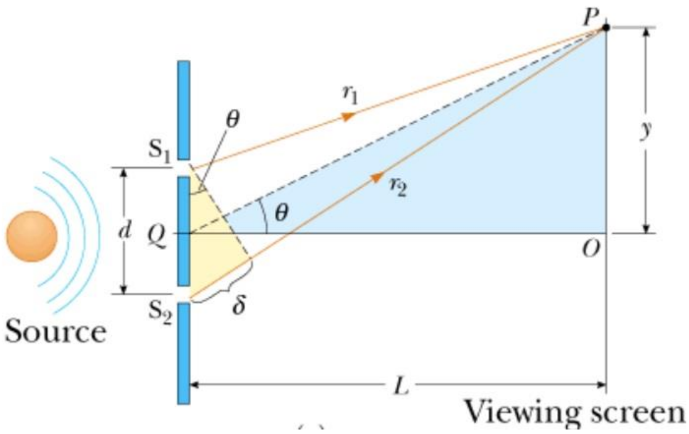
(a) What is the separation between the slits?

(b) What is the path difference from the diffraction grating to the fourth maximum, measured in meters?

(c) A new laser whose wavelength is $\lambda_b = 6.3 \times 10^{-7}$ m shines through the slits and both its fourth maximums are labeled. What is the distance from the fourth order maximum using λ_a and the closest fourth maximum using λ_b?

Solution:

Consider the light rays from the two coherent point sources made from *infinitesimal* slits a distance d apart. We assume that the sources are emitting monochromatic light of wavelength λ. The rays are emitted in all forward directions, but let us concentrate on only the rays that are emitted in a direction θ toward a distant screen (θ measured from the normal to the screen, diagram below) . One of these rays has further to travel to reach the screen, and the *path difference* is given by $d\sin\theta$. If this path difference is exactly one wavelength λ or an integer number of wavelengths, then the two waves arrive at the screen in phase and there is constructive interference, resulting in a bright area on the screen. If the path difference is $\frac{1}{2}\lambda$, or $\frac{3}{2}\lambda$, etc., then there is destructive interference, resulting in a dark area on the screen.



$$\delta = r_2 - r_1 = d\sin\theta,$$

$$\left.\begin{aligned} &Bright: \ d\sin\theta = m\lambda \\ &Dark: \ d\sin\theta = (m+\tfrac{1}{2})\lambda. \end{aligned}\right\} m = 0, \pm 1, \pm 2..$$

$d \gg \lambda, \qquad \sin\theta \approx \tan\theta$

$y = L\tan\theta \ \approx L\sin\theta \approx \tan\theta$

$$y_{bright} = \frac{\lambda L}{d} m$$

$$y_{dark} = \frac{\lambda L}{d}\left(m + \frac{1}{2}\right)$$

Answer:

A laser whose wavelength is $\lambda_a = 4.7 \times 10^{-7}$ m shines through a diffraction grating. The light hits a screen L = 3 m away and fourth order maximum is detected 0.52 m on either side of the central maximum.

(a) What is the separation between the slits?

$$y_{bright} = \frac{\lambda_a L}{d} m$$

Thus,

$$d = \frac{\lambda_a L}{y_{bright}} m = \frac{4.7 * 10^{-7} * 3}{0.52} * 4 = 108.46 * 10^{-7} = 1.08 * 10^{-5} \text{ m}$$

(b) What is the path difference from the diffraction grating to the fourth maximum, measured in meters?

The pass difference is

$$\delta = r_2 - r_1 = d \sin\theta$$

$$\sin\theta \approx \tan\theta \ = \frac{y}{L}$$

Hence,

$$\delta = \frac{dy}{L} = \frac{1.08 * 10^{-5} * 0.52}{3} = 1.872 * 10^{-6} \text{ m}$$

(c) A new laser whose wavelength is $\lambda_b = 6.3 \times 10^{-7}$ m shines through the slits and both its fourth maximums are labeled. What is the distance from the fourth order maximum using λ_a and the closest fourth maximum using λ_b?

$$y_b = \frac{\lambda_b L}{d} m = \frac{6.3 * 10^{-7} * 3}{1.08 * 10^{-5}} * 4 = 0.7 \text{ m}$$

$$\Delta y = y_b - y_a = 0.7 - 0.52 = 0.18 \text{ m}$$