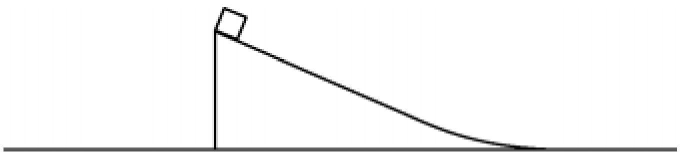
**Answer on Question #48970-Physics-Mechanics-Kinematics-Dynamics**

The figure above shows part of a system consisting of a block at the top of an inclined plane that rests on a table, which is located on Earth. The block and plane are at rest when the block is released. In trial 1 there is no friction between the block and the plane or between the plane and the table. In trial 2 the plane is fixed to the table so it cannot move, but there is still no friction between the block and the plane.

(a) Indicate whether the speed of the block relative to the table when the block reaches the bottom of the plane is greater in trial 1 or trial 2. Justify your answer in a clear, coherent, paragraph-length explanation.

(b) Assuming the body has mass M and the plane has mass 3M and angle 30o, find the ratio of the speeds of the block at the base of the plane, just before entering the curved area of the plane, in the 2 cases ( trial 1 / trial 2).

**Solution**



Assume the body has mass $m$ and the plane has mass $M$, the angle be $\alpha$.

(a) In trial 1 the weight of the body has parallel and perpendicular components to a plane;

$$W_{\perp} = W\cos\alpha\,, W_{\parallel} = W\sin\alpha.$$

A plane is moving to the left by the force $W_{\perp}\sin\alpha = W\sin\alpha\cos\alpha$.

The acceleration of plane is

$$a_p = \frac{W\sin\alpha\cos\alpha}{M}.$$

Vertical acceleration of the body is

$$a_y = \frac{W_{\parallel}\sin\alpha}{m} = \frac{W\sin^2\alpha}{m}.$$

But the horizontal acceleration of the body relative to the table is

$$a_x = \frac{W\sin\alpha\cos\alpha}{m} - \frac{W\sin\alpha\cos\alpha}{M}.$$

The magnitude of acceleration of the body relative to the table is

$$a_1 = \sqrt{a_x^2 + a_y^2} = \frac{W}{m}\sqrt{\sin^4\alpha + \sin^2\alpha\cos^2\alpha\left(1 - \frac{m}{M}\right)^2} = \frac{W\sin\alpha}{m}\sqrt{\sin^2\alpha + \cos^2\alpha\left(1 - \frac{m}{M}\right)^2}.$$

The speed of the block relative to the table when the block reaches the bottom of the plane is

$$v_1 = a_1 t,$$

where $t = \sqrt{\frac{2l}{a_\parallel}}$, $a_\parallel = \frac{W \sin\alpha}{m}$.

In trial 2 the weight of the body has parallel and perpendicular components to a plane;

$$W_\perp = W\cos\alpha\,, W_\parallel = W\sin\alpha.$$

The magnitude of acceleration of the body relative to the table is

$$a_\parallel = \frac{W\sin\alpha}{m}.$$

The speed of the block relative to the table when the block reaches the bottom of the plane is

$$v_2 = a_\parallel t,$$

where $t = \sqrt{\frac{2l}{a_\parallel}}$.

The ratio of the speeds of the block at the base of the plane is

$$\frac{v_2}{v_1} = \frac{a_\parallel t}{a_1 t} = \frac{\frac{W_\parallel \sin\alpha}{m}}{\frac{W\sin\alpha}{m}\sqrt{\sin^2\alpha + \cos^2\alpha\left(1-\frac{m}{M}\right)^2}} = \frac{1}{\sqrt{\sin^2\alpha + \cos^2\alpha\left(1-\frac{m}{M}\right)^2}} > 1.$$

Because

$$\sqrt{\sin^2\alpha + \cos^2\alpha\left(1-\frac{m}{M}\right)^2} < \sqrt{\sin^2\alpha + \cos^2\alpha} = \sqrt{1} = 1.$$

So, the speed of the block relative to the table when the block reaches the bottom of the plane is greater in trial 2.

(b)

$$\frac{v_1}{v_2} = \sqrt{\sin^2\alpha + \cos^2\alpha\left(1-\frac{m}{M}\right)^2} = \sqrt{\sin^2 30 + \cos^2 30\left(1-\frac{M}{3M}\right)^2} = \sqrt{\frac{1}{4}+\frac{3}{4}\left(\frac{2}{3}\right)^2} = \sqrt{\frac{1}{4}+\frac{1}{3}} = \sqrt{\frac{7}{12}}.$$