1. A bucket of oil is released at the top of a building to hit a person. The mass of the bucket with the oil is 2.5kg and it is attached to a massless string that is wrapped around a cylindrical drum. As the bucket falls, the drum rotates and you may assume that there is no slippage. If the drum has a radius of .85m and a mass of 12.5kg, what is the angular speed of the drum after the oil has fallen a distance of 4.5m? Use the conservation of energy.

$m = 2.5\,kg$
$r = 0.85\,m$
$M = 12.5\,kg$
$h = 4.5\,m$
$\omega - ?$

Solution.

Let write the moment equation for the drum rotation: $I \cdot \beta = F \cdot r$,

where $I = \frac{Mr^2}{2}$ is the moment f inertia of a uniform drum during its rotation, the force is the bucket weight $F = mg$.

So, the angular acceleration of the drum is constant: $\beta = \frac{2mg}{Mr}$.

The height h is the whole path of the drum's edge: $h = 2\pi r \cdot \alpha = 2\pi r \cdot \frac{\omega^2}{2\beta}$,

where α is total angle of the rotation, ω is final angular speed.

$h = 2\pi r \cdot \frac{\omega^2}{2 \cdot \frac{2mg}{Mr}}$. Thus, $\boxed{\omega = \frac{1}{r}\sqrt{\frac{mg\,h}{\pi M}}}$.

Let check the dimension: $[\omega] = \frac{1}{m}\sqrt{\frac{kg \cdot \frac{m}{s^2} \cdot m}{kg}} = \frac{rad}{s}$.

Let evaluate the quantity: $\omega = \frac{1}{0.85}\sqrt{\frac{2.5 \cdot 9.8 \cdot 4.5}{3.14 \cdot 12.5}} = 1.97\left(\frac{rad}{s}\right)$.

Answer: $1.97\frac{rad}{s}$.