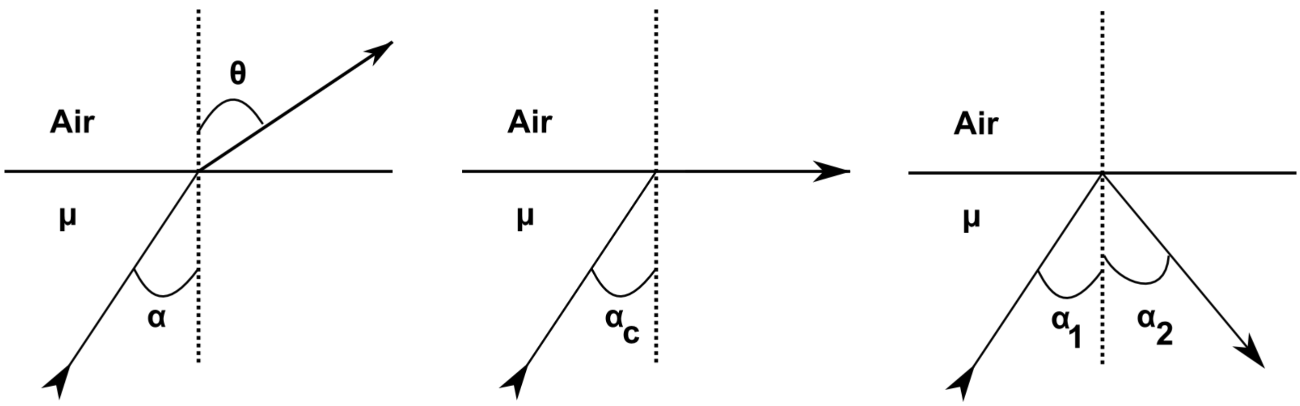
Answer on Question 48846, Physics, Optics

Question:

A ray of light is incident at an angle (α) on a planar end of a transparent cylindrical glass rod of refractive index (μ). Determine the least value of μ so that light entering the rod does not emerge from the curved surface of the rod irrespective of the angle α.

Solution:



Let α be the angle of incidence, and θ be the angle of refraction. In order to light does not emerge from the curved surface of the rod we need to satisfy the condition of total internal reflection. When the angle of incidence exceeds the critical angle the total internal reflection occurs, we can see it on the picture. So, let's write the relationship between critical angle and refractive index, it's looks like: $\mu = \frac{1}{\sin \alpha_c}$, where α_c is the critical angle. Then we have: $\sin(90^\circ - \theta) > \sin \alpha_c$. From the trigonometric identities we know, that $\sin(90^\circ - \theta) = \cos \theta$, so we can rearrange expression: $\cos \theta > \frac{1}{\mu} (1)$.

From Snell's law $\sin \theta = \frac{\sin \alpha}{\mu}$. Again, we use the trigonometric identities and we can write: $\cos \theta = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - \left(\frac{1}{\mu^2}\right) \sin^2 \alpha}\ (2)$. From consideration of formulas (1) and (2) we can obtain:

$$\sqrt{1-\frac{1}{\mu^2}\sin^2\alpha} > \frac{1}{\mu},$$

$$\sqrt{\mu^2-\sin^2\alpha} > 1.$$

So, in order to light entering the rod does not emerge from the curved surface of the rod irrespective of the angle α, we must satisfy the condition $\alpha = 90^\circ$. In this case we have: $\sqrt{\mu^2-1} > 1$, from which we find that $\mu > \sqrt{2}$.

Answer:

Therefore, the least value of μ is $\sqrt{2}$.