Answer on Question #48601 – Physics – Mechanics | Kinematics | Dynamics

1. When a particle is projected at some angles to the horizontal, it has a range R and time of flight t1. If the same particle is projected with same speed at some other angle to have the same range its time of flight is t2, then,

1. t1 + t2 = 2R/g; 2. t1 - t2 = R/g; 3. t1t2 = 2R/g; 4. t1t2 = R/g

R
t_1, t_2
$t_1, t_2 - ?$

Solution.

Let write the kinematic equations of a particle's motion. If the center of coordinate system is in its initial position, then the coordinates depend on time as:

$$\begin{cases} x = v_0 \cos\alpha \cdot t \\ y = v_0 \sin\alpha \cdot t - \dfrac{gt^2}{2} \end{cases}.$$

Here X-axis is directed towards the motion and Y-axis is directed up.

The time of falling can be found from the condition $y = 0$:

$$v_0 \sin\alpha \cdot t - \frac{gt^2}{2} = 0, \quad t_0 = \frac{2v_0}{g}\sin\alpha .$$

So, the range is

$$R = x(t_0) = v_0 \cos\alpha \cdot t_0 = v_0\sqrt{1-\sin^2\alpha} \cdot t_0 = v_0\sqrt{1-\left(\frac{gt_0}{2v_0}\right)^2} \cdot t_0 = \frac{t_0}{2}\sqrt{4v_0^2 - g^2 t_0^2} .$$

Thus, $R^2 = \dfrac{t_0^2}{4}\left(4v_0^2 - g^2 t_0^2\right), \quad v_0^2 = \dfrac{R^2}{t_0^2} + \dfrac{g^2 t_0^2}{4}$.

One can write that $v_0^2 = \dfrac{R^2}{t_1^2} + \dfrac{g^2 t_1^2}{4} = \dfrac{R^2}{t_2^2} + \dfrac{g^2 t_2^2}{4}$.

Let transform the last equation.

$$\frac{R^2}{t_1^2} - \frac{R^2}{t_2^2} = \frac{g^2 t_2^2}{4} - \frac{g^2 t_1^2}{4}, \quad \frac{R^2\left(t_2^2 - t_1^2\right)}{t_1^2 t_2^2} = \frac{g^2\left(t_2^2 - t_1^2\right)}{4}, \quad t_1 t_2 = \frac{2R}{g}.$$

Answer: 3.