Answer on Question #48422-Physics-Mechanics-Kinematics-Dynamics

Sunil played a stroke and hits a ball with a velocity of 20 m/s at an angle of elevation 45 degree. Kapil standing near the boundary along the line of stroke, 50m away from the batsman, starts running to catch the ball at the instant the ball is hit. He managed to run at constant acceleration and at the last moment he took the catch 2 m above the level of the stroke point. Find

1. the acceleration with which kapil must have run.

2. velocity of the ball when caught by fielder

3. velocity of fielder while catching the ball.

Solution

This is a problem about a projectile motion.

The accelerations in the x and y directions can be integrated to solve for the components of velocity at any time t, as follows:

$$a_x = 0; a_y = -g.$$

The accelerations in the x and y directions can be integrated to solve for the components of velocity at any time t, as follows:

$$v_x = v_0 \cos\theta, v_y = v_0 \sin\theta - gt.$$

The magnitude of the velocity (under the Pythagorean theorem):

$$v = \sqrt{v_x^2 + v_y^2}.$$

At any time t, the projectile's horizontal and vertical displacement:

$$x = v_0 t \cos\theta; y = v_0 t \sin\theta - \frac{gt^2}{2}.$$

We need to find the time of flight of the ball.

$$y(t_f) = v_0 t_f \sin\theta - \frac{g t_f^2}{2}$$

or

$$2\,m = 20\frac{m}{s}\cdot\ t_f \sin 45 - \frac{10\frac{m}{s^2}\cdot t_f^2}{2} \to 5t_f^2 - \frac{20}{\sqrt{2}}t_f + 2 = 0.$$

$$D = \left(\frac{20}{\sqrt{2}}\right)^2 - 4\cdot 5\cdot 2 = 160.$$

$$t_f = \frac{\frac{20}{\sqrt{2}} \pm 4\sqrt{10}}{2\cdot 5} = \sqrt{2} \pm \frac{4}{\sqrt{10}}.$$

The time $\sqrt{2} - \frac{4}{\sqrt{10}} \approx 0.14\ s$ doesn't fulfill the conditions, so

$$t_f = \sqrt{2} + \frac{4}{\sqrt{10}} \approx 2.7\ s.$$

The velocity of the ball when caught by fielder is

$$v_y(t_f) = 20\frac{m}{s} \cdot \sin 45 - 10\frac{m}{s^2} \cdot 2.7\ s;\ v_x = 20\frac{m}{s} \cdot \cos 45$$

$$v_b = \sqrt{\left(20 \cdot \frac{1}{\sqrt{2}} - 10 \cdot 2.7\right)^2 + \left(20 \cdot \frac{1}{\sqrt{2}}\right)^2} = 19.1\frac{m}{s}.$$

The projectile's horizontal and vertical displacement at time t_f:

$$x = 20 \cdot \frac{1}{\sqrt{2}} \cdot 2.7 = 38.2\ m.$$

So, the distance travelled by the catcher:

$$x_c = 50 - 38.2 = 11.8\ m.$$

He managed to run at constant acceleration, so

$$x_c = \frac{1}{2}at_f^2.$$

The acceleration with which kapil must have run is

$$a = \frac{2x_c}{t_f^2} = \frac{2 \cdot 11.8}{2.7^2} = 3.2\frac{m}{s^2}.$$

Velocity of fielder while catching the ball is

$$v_c = at_f = 3.2 \cdot 2.7 = 8.6\frac{m}{s}.$$

Answer: 1. $3.2\frac{m}{s^2}$; 2. $19.1\frac{m}{s}$; 3. $8.6\frac{m}{s}$.