

1. A crate is pushed off the edge of a building which is 160 m tall. The crate lands 35 m from the base of the building. How fast was the crate pushed, and how fast will it be moving when it hits the ground?

$$\begin{array}{l} h = 160 \text{ m} \\ l = 35 \text{ m} \\ v_0, v_1 - ? \end{array}$$

Solution.

We assume that the initial velocity of the crate was horizontal.

Let write the kinematic equations of motion of the crate. If the center of coordinate system is in its initial position, then the coordinates depend on time as:

$$\begin{cases} x = v_0 \cdot t \\ y = -\frac{gt^2}{2} \end{cases}$$

Here X -axis is directed towards the motion and Y -axis is directed up.

When the crate lands, then $y = -h$. One can find the correspondent time:

$$-h = -\frac{gt^2}{2}, \quad t_1 = \sqrt{\frac{2h}{g}}.$$

Then, we can find the initial velocity: $v_0 = \frac{x(t_1)}{t_1} = l : \sqrt{\frac{2h}{g}}, \quad \boxed{v_0 = l \sqrt{\frac{g}{2h}}}$.

The components of the velocity of the crate are: $\begin{cases} v_x = v_0 \\ v_y = -gt \end{cases}$.

So, the velocity of the moment of the landing:

$$v_1 = \sqrt{(v_x(t_1))^2 + (v_y(t_1))^2} = \sqrt{v_0^2 + g^2 t_1^2} = \sqrt{l^2 \cdot \frac{g}{2h} + g^2 \cdot \frac{2h}{g}}, \quad \boxed{v_1 = \sqrt{g \left(\frac{l^2}{2h} + 2h \right)}}$$

Let check the dimension: $[v_0] = m \cdot \sqrt{\frac{m/s^2}{m}} = \frac{m}{s}, \quad [v_1] = \sqrt{\frac{m}{s^2} \cdot m} = \frac{m}{s}$.

Let evaluate the quantities:

$$v_0 = 35 \cdot \sqrt{\frac{9.81}{2 \cdot 160}} = 6.13 \left(\frac{m}{s} \right), \quad v_1 = \sqrt{9.81 \cdot \left(\frac{35^2}{2 \cdot 160} + 2 \cdot 160 \right)} = 56.4 \left(\frac{m}{s} \right).$$

Answer: $6.13 \frac{m}{s}, \quad 56.4 \frac{m}{s}$.