Answer on Question #48256 – Physics - Mechanics | Kinematics | Dynamics

What is scalar product or dot product?
What is vector product or cross product ?

Solution:

Dot product

The dot product can be defined for two vectors $\bar{x}$ and $\bar{y}$ by

$$\bar{x} \cdot \bar{y} = |x| \cdot |y| \cdot \cos\theta$$

where θ is the angle between the vectors and $|x|$ is the norm. It follows immediately that $\bar{x} \cdot \bar{y} = 0$ if $\bar{x}$ is perpendicular to $\bar{y}$.

The dot product therefore has the geometric interpretation as the length of the projection of $\bar{x}$ onto the unit vector $\hat{y}$ when the two vectors are placed so that their tails coincide.

Cross product

For vectors $u = (u_x, u_y, u_z)$ and $v = (v_x, v_y, v_z)$ in $\mathbb{R}^3$, the cross product in is defined by

$$u \times v = \hat{x}(u_y v_z - u_z v_y) - \hat{y}(u_x v_z - u_z v_x) - \hat{z}(u_x v_y - u_y v_x) =$$
$$= \hat{x}(u_y v_z - u_z v_y) + \hat{y}(u_z v_x - u_x v_z) + \hat{z}(u_x v_y - u_y v_x)$$

where $(\hat{x}, \hat{y}, \hat{z})$ is a right-handed, i.e., positively oriented, orthonormal basis. This can be written in a shorthand notation that takes the form of a determinant

$$u \times v = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix}$$

where $\hat{x}, \hat{y}$ and $\hat{z}$ are unit vectors. Here, $u \times v$ is always perpendicular to both u and v, with the orientation determined by the right-hand rule.