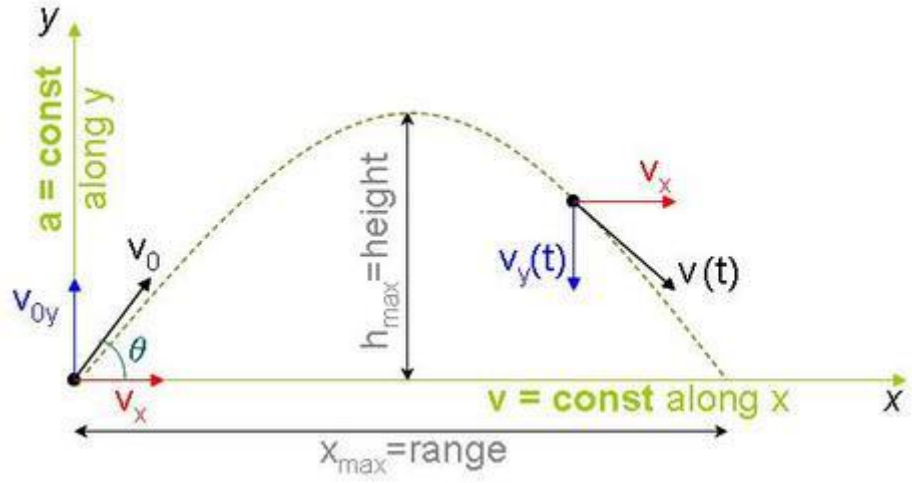
Answer on Question #47944, Physics, Other

You fire a projectile from one building to another building. The building you fire from is 54 meters tall, the building you fire to is 76 meters tall. The buildings are 1485 meters apart. If you want your maximum height to be 100 meters, what is the angle that you must launch at? What is the angle that the projectile will land at?

Solution:



Projectile motion is a form of motion in which an object or particle (called a projectile) is thrown near the earth's surface, and it moves along a curved path under the action of gravity only.

In projectile motion, the horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other.

The horizontal component of the velocity of the object remains unchanged throughout the motion. The vertical component of the velocity increases linearly, because the acceleration due to gravity is constant (g=9.81 m/s^2).

Equations related to trajectory motion are given by

$$\text{Horizontal distance,} \quad x_{max} = v_{0x}t$$

$$\text{Vertical distance,} \quad y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

$$\text{Horizontal range,} \quad R = x_{max} = \frac{v_0^2 \sin 2\theta}{g}$$

$$\text{Maximum height reached,} \quad H = \frac{v_0^2 \sin^2 \theta}{2g}$$

where v_0 is the initial velocity.

In our case:

$$H = 100 - 54 = 46 \text{ m}$$

Thus, from fourth equation

$$v_{0y} = v_0 \sin\theta = \sqrt{2gH} = \sqrt{2 * 9.81 * 46} = 30.04 \text{ m/s}$$

From second equation time of flight

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

$$76 = 54 + 30.04t - 4.905t^2$$

$$4.905t^2 - 30.04t + 22 = 0$$

Solutions:

$$t_1 = 0.85 \text{ s}$$
$$t_2 = 5.274 \text{ s}$$

Horizontal distance:

$$x = v_{0x}t = v_0 \cos\theta \, t$$
$$v_0 \cos\theta = \frac{x}{t_2} = \frac{1485}{5.274} = 281.57 \text{ m/s}$$

So we have

$$v_0 \sin\theta = 30.04 \text{ m/s}$$
$$v_0 \cos\theta = 281.57 \text{ m/s}$$
$$\tan\theta = \frac{30.04}{281.57} = 0.106687$$

The angle that you must launch at is

$$\theta = \tan^{-1} 0.106687 = 6.09°$$

The angle that the projectile will land at

$$\theta_2 = \tan^{-1}\left(\frac{v_y}{v_{0x}}\right) = \tan^{-1}\left(\frac{v_{0y} - gt_2}{v_{0x}}\right)$$
$$\theta_2 = \tan^{-1}\left(\frac{30.04 - 9.81 * 5.274}{281.57}\right) = -4.407°$$

Answer: $\theta = 6°, \theta_2 = -4.4°$