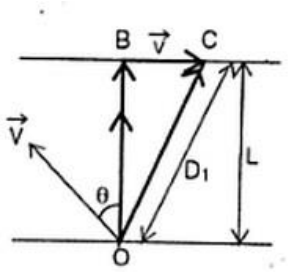
Answer on Question #47673-Physics-Mechanics-Kinematics-Dynamics

To cross a river in the shortest time why the total distance crossed by the boat is more than the boat cross the river across to the shortest distance? To cross across to the shortest distance it takes more time than the time needed to cross the river in shortest time and this time the distance is more than the shortest distance. If we think generally to cross long distance it takes long time. But in this case why it is different?

Solution

a.



Suppose the boat starts in travelling in a direction making an angle θ with OB as shown in the figure. If L is the width of the river, time taken by the boat to cross the river will be

$$t = \frac{L}{V\cos\theta} \ (as\ c\text{omponent of V along OB will be } V\cos\theta).$$

The time will be minimum when $\cos\theta = max = 1 \rightarrow \theta = 0°$ i.e., <u>to cross the river in shortest time, the boat should travel perpendicular to the flow.</u>

In above case:

Time taken to cross the river $t_1 = \frac{L}{V}$.

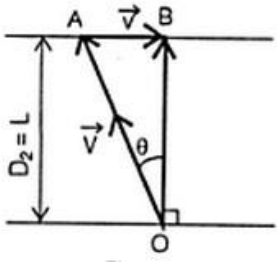
During this time the flow of water will take the boat from B to C such that

$$BC = vt_1 = \frac{vL}{V}.$$

The resultant displacement of the boat from starting point

$$\mathrm{OC} = \mathrm{D}_1 = \sqrt{L^2 + \left(\frac{vL}{V}\right)^2} = L\sqrt{1 + \left(\frac{v}{V}\right)^2}.$$

b.



Because the shortest distance between two banks is the perpendicular distance between them i.e., L, so in the order to cross the river along the shortest path $\mathrm{OB} = L$ the boat should travel at an angle θ to OB such that the horizontal component of its velocity balance the speed of flow i.e.,

$$V\sin\theta = v \rightarrow \sin\theta = \frac{v}{V} \rightarrow \theta = \sin^{-1}\frac{v}{V}$$

i.e., to cross the river along shortest path, the boat should travel at an angle $(90 + \theta)$ to the direction of flow, with $\theta = \sin^{-1}\frac{v}{V}$.

In this case:

Time taken to cross the river $t_2 = \frac{L}{V\cos\theta}\left(= \frac{t_1}{\cos\theta} > t_1\right)$.

Resultant displacement of the boat $D_2 = L < D_1$

i.e., when the path in shortest time is not least and when time is least path is not shortest.