Answer on Question #47505-Physics-Mechanics-Kinematics-Dynamics

Two bodies A and B of equal masses m lie on a smooth frictionless horizontal table. They are initially separated by a distance $2l$ and are connected by a thread of length $2l$. The thread is pulled at its midpoint O by a constant force F at right angle to AB. Find the acceleration of A and B in the direction at right angles to the direction of F as a function of the displacement (x) of the midpoint O

Solution

When the thread is pulled the tensions T of the halves of the thread acts on bodies A and B. In y-direction (at right angle to AB):

$$F = 2T\cos\alpha,$$

where $\cos\alpha = \frac{x}{l}$.

In z-direction (at right angles to the direction of F):

$$ma_z = T\sin\alpha.$$

The acceleration of A and B in the direction at right angles to the direction of F is

$$a_z = \frac{T}{m}\sin\alpha = \frac{F}{2m}\frac{\sin\alpha}{\cos\alpha} = \frac{F}{2m}\frac{\sqrt{1-\left(\frac{x}{l}\right)^2}}{\frac{x}{l}} = \frac{F}{2m}\sqrt{\left(\frac{l}{x}\right)^2 - 1}, when\ x \le l\ and\ 0\ when\ x > l.$$

Answer: $\begin{cases} \frac{F}{2m}\sqrt{\left(\frac{l}{x}\right)^2 - 1}, & \boldsymbol{x \le l} \\ \mathbf{0}, & \boldsymbol{x > l}. \end{cases}$