Answer on Question #47407, Physics, Mechanics | Kinematics | Dynamics

A football quarterback throws a pass at an angle of 38.1°. He releases the pass 3.50 m behind the line of scrimmage. His receiver left the line of scrimmage 2.8 s earlier, going straight down-field at a constant speed of 7.50 m/s. With what speed must the quarterback throw the ball so that the pass lands gently in the receiver's hands without the receiver breaking stride? Assume that the ball is released at the same height it is caught and that the receiver is straight downfield from the quarterback at the time of release. (Ignore any effects due to air resistance.)

Solution:



Projectile motion is a form of motion in which an object or particle (called a projectile) is thrown near the earth's surface, and it moves along a curved path under the action of gravity only.

In projectile motion, the horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other.

The horizontal component of the velocity of the object remains unchanged throughout the motion. The vertical component of the velocity increases linearly, because the acceleration due to gravity is constant (g = 9.81 m/s^2).

The initial distance between quarterback and receiver is

$$d_0 = v_1 t_1 + d_2 = 7.50 \cdot 2.8 + 3.50 = 24.5 \text{ m}$$

Time of ball flight

$$t = \frac{2v_0 \sin \theta}{g}$$

Horizontal distance, $d = v_{0x}t$

The distance covered by receiver

$$d_1 = v_1 t = d - d_0$$

Thus,

$$v_1 \frac{2v_0 \sin \theta}{g} = v_0 \cos \theta \frac{2v_0 \sin \theta}{g} - d_0$$

So,

$$v_0^2 \frac{\sin 2\theta}{g} - v_0 \frac{2v_1 \sin \theta}{g} - d_0 = 0$$

$$v_0^2 \frac{\sin 2 \cdot 38.1^\circ}{9.81} - v_0 \frac{2 \cdot 7.50 \cdot \sin 38.1^\circ}{g} - 24.5 = 0$$

 $0.09899v_0^2 - 0.9435v_0 - 24.5 = 0$

Solution of this quadratic equation gives us

$$v_0 = 21.2037 \text{ m/s}$$

Answer: $v_0 = 21.2 \text{ m/s}$