

**Answer on Question #46995 – Math – Calculus**

**Question.** Discuss the continuity of the function  $f$ , defined by

$$f(x) = \begin{cases} x^2 - 1, & x \leq 1, \\ 1 - 1/x, & x \geq 1. \end{cases}$$

**Solution.** 1) Suppose  $x < 1$ . Then  $f(x) = x^2 - 1$  is a polynomial, and therefore it is continuous at each such  $x$ .

2) Suppose  $x = 1$ . Then left limit of  $f$  at  $x = 1$  is equal to

$$\lim_{x \rightarrow 1-0} f(x) = \lim_{x \rightarrow 1-0} x^2 - 1 = 1^2 - 1 = 1 - 1 = 0,$$

and the right limit is

$$\lim_{x \rightarrow 1+0} f(x) = \lim_{x \rightarrow 1+0} 1 - 1/x = 1 - 1/1 = 1 - 1 = 0.$$

Thus left and right limits of  $f$  at  $x = 1$  coincide, and therefore  $f$  is continuous at  $x = 1$ .

3) Finally, let  $x > 1$ . Then  $f(x) = 1 - 1/x$ . Since  $x \neq 0$ , this function is continuous at all such  $x$ .

Thus  $f$  is continuous at all  $x \in \mathbb{R}$ .

**Answer.**  $f$  is continuous at all  $x \in \mathbb{R}$ .