

Answer on Question #46809, Physics, Mechanics | Kinematics | Dynamics

A solid sphere and a disc of same radius and mass are rolled down a rough inclined plane. the ratio of time taken by each to reach bottom of plane is?

Next, if they are rolled in smooth inclined plane, then ratio of time taken?

a. Rough inclined plane – rolling:

Using energy conservation law for a sphere:

$$E_P = E_K$$

$$mgh = \frac{I\omega^2}{2} + \frac{mv^2}{2} = \frac{2}{5} \frac{mr^2\omega^2}{2} + \frac{mv^2}{2} = \frac{2}{5} \frac{mv^2}{2} + \frac{mv^2}{2} = \frac{7mv^2}{10}$$

Where I - moment of inertia, ω – angular frequency, v – linear speed

$$v_s = \sqrt{\frac{10gh}{7}}$$

Using energy conservation law for a disc:

$$E_P = E_K$$

$$mgh = \frac{I\omega^2}{2} + \frac{mv^2}{2} = \frac{1}{2} \frac{mr^2\omega^2}{2} + \frac{mv^2}{2} = \frac{1}{2} \frac{mv^2}{2} + \frac{mv^2}{2} = \frac{3mv^2}{4}$$

Where I - moment of inertia, ω – angular frequency, v – linear speed

$$v_c = \sqrt{\frac{4gh}{3}}$$

From the other hand:

$$s = \frac{at^2}{2} = \frac{vt}{2} \rightarrow t = \frac{2s}{v}$$

Thus, the ratio of time taken by each to reach bottom of plane is:

$$\frac{t_s}{t_c} = \frac{\frac{2s}{v_s}}{\frac{2s}{v_c}} = \frac{v_c}{v_s} = \frac{\sqrt{\frac{4gh}{3}}}{\sqrt{\frac{10gh}{7}}} = \sqrt{\frac{28}{30}} = \sqrt{\frac{14}{15}} \approx 0.97$$

b. If they are rolled in smooth inclined plane they will slide without rolling and the times will be equal

Answer: a. The ratio of time taken by each to reach bottom of plane is $\frac{t_s}{t_c} \approx 0.97$

b. The ratio of time taken by each to reach bottom of plane is $\frac{t_s}{t_c} = 1$