

Answer on Question #46809, Physics, Mechanics | Kinematics | Dynamics

A solid sphere and a disc of same radius and mass are rolled down a rough inclined plane. the ratio of time taken by each to reach bottom of plane is?

Next, if they are rolled in smooth inclined plane, then ratio of time taken?

- a. Rough inclined plane – rolling:

Using energy conservation law for a sphere:

$$E_P = E_K$$
$$mgh = \frac{I\omega^2}{2} + \frac{mv^2}{2} = \frac{2}{5} \frac{mr^2\omega^2}{2} + \frac{mv^2}{2} = \frac{2}{5} \frac{mv^2}{2} + \frac{mv^2}{2} = \frac{7mv^2}{10}$$

Where I - moment of inertia, ω – angular frequency, v – linear speed

$$v_s = \sqrt{\frac{10gh}{7}}$$

Using energy conservation law for a disc:

$$E_P = E_K$$
$$mgh = \frac{I\omega^2}{2} + \frac{mv^2}{2} = \frac{1}{2} \frac{mr^2\omega^2}{2} + \frac{mv^2}{2} = \frac{1}{2} \frac{mv^2}{2} + \frac{mv^2}{2} = \frac{3mv^2}{4}$$

Where I - moment of inertia, ω – angular frequency, v – linear speed

$$v_c = \sqrt{\frac{4gh}{3}}$$

From the other hand:

$$s = \frac{at^2}{2} = \frac{vt}{2} \rightarrow t = \frac{2s}{v}$$

Thus, the ratio of time taken by each to reach bottom of plane is:

$$\frac{t_s}{t_c} = \frac{\frac{2s}{v_s}}{\frac{2s}{v_c}} = \frac{v_c}{v_s} = \frac{\sqrt{\frac{4gh}{3}}}{\sqrt{\frac{10gh}{7}}} = \sqrt{\frac{28}{30}} = \sqrt{\frac{14}{15}} \approx 0.97$$

- b. If they are rolled in smooth inclined plane they will slide without rolling and the times will be equal

Answer: a. The ratio of time taken by each to reach bottom of plane is $\frac{t_s}{t_c} \approx 0.97$

b. The ratio of time taken by each to reach bottom of plane is $\frac{t_s}{t_c} = 1$