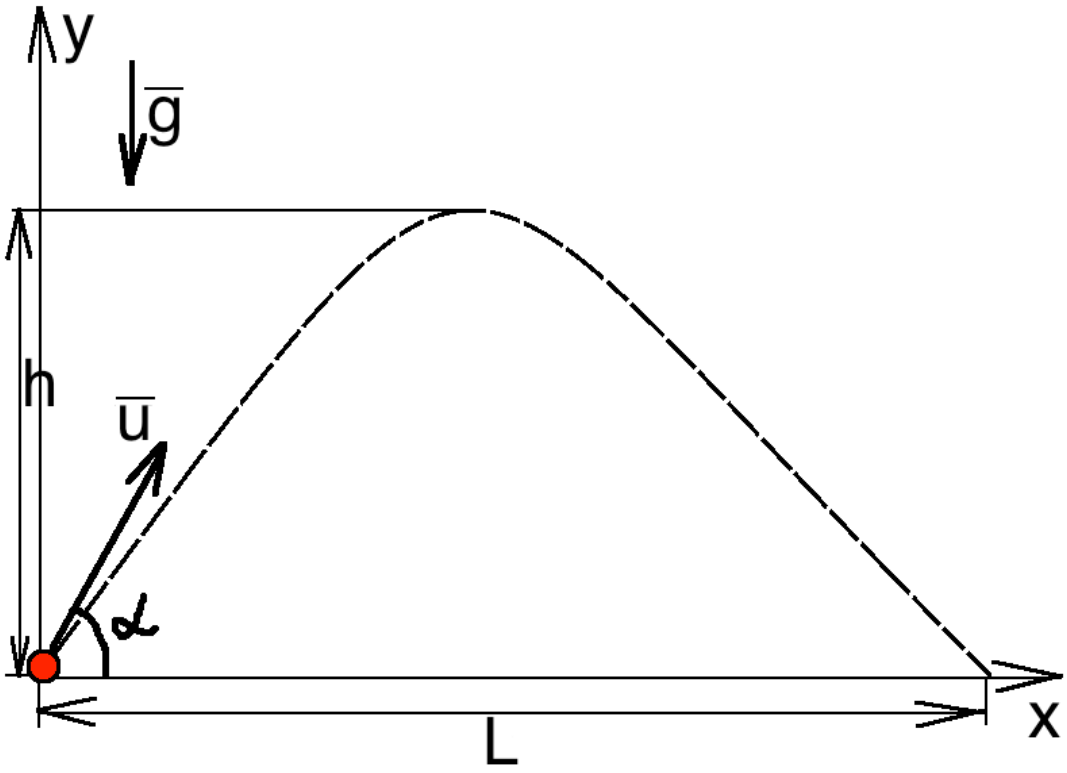
Answer on Question #46556 – Physics - Mechanics | Kinematics | Dynamics

Find the range of a ball which was when projected with initial velocity 29.4 m/s it just passes over a 4.9 m high pole.

Solution:

$u = 29.4\frac{m}{s} - \text{initial speed of the ball;}$
$h = 4.9m - \text{maximum height that ball riches}$
$L - \text{range of a ball;}$



Equation of the motion for body, thrown at angle α: (L - maximum horizontal range of this body) ($t - \text{time of the flight}$)

$u_x = u\cos\alpha;\ u_y = u\sin\alpha;$

$x: L = ut\cos\alpha \qquad (1)$

$y: 0 = ut\sin\alpha - \frac{gt^2}{2}$

$u\sin\alpha = \frac{gt}{2}$

$t = \frac{2u\sin\alpha}{g} \qquad (2)$

(2) in (1):

$L = u\frac{2u\sin\alpha}{g}\cos\alpha = \frac{2u^2\sin\alpha\cos\alpha}{g} \qquad (3)$

Maximum height: the time taken to reach the maximum height is equal to half of the time of flight:

$$t_1 = \frac{t}{2} = \frac{u \sin\alpha}{g}$$

y: (half of the flight): $h_1 = ut_1 \sin\alpha - \frac{g{t_1}^2}{2}$

$$h = ut_1 \sin\alpha - \frac{g{t_1}^2}{2}$$

$$h = u\frac{V \sin\alpha}{g} \sin\alpha - \frac{g\left(\frac{u \sin\alpha}{g}\right)^2}{2} = \frac{u^2 \sin^2\alpha}{2g} \qquad (4)$$

from (4)

$$\sin^2\alpha = \frac{2gh}{u^2} \qquad \sin\alpha = \sqrt{\frac{2gh}{u^2}} \qquad (5)$$

Pythagorean Identity

$$\sin^2\alpha + \cos^2\alpha = 1$$
$$\cos^2\alpha = 1 - \sin^2\alpha \quad (6)$$

(5)in(6):

$$\cos^2\alpha = 1 - \frac{2gh}{u^2} \qquad \cos\alpha = \sqrt{1 - \frac{2gh}{u^2}} \qquad (7)$$

(5)and(7)in(3):

$$L = \frac{2u^2 \sin\alpha \cos\alpha}{g} = \frac{2u^2\sqrt{\left(\frac{2gh}{u^2}\right)\left(1 - \frac{2gh}{u^2}\right)}}{g} = \frac{2\sqrt{(2gh)(u^2 - 2gh)}}{g} =$$
$$= \frac{2\sqrt{\left(2 \cdot 9.8\frac{m}{s^2} \cdot 4.9m\right)\left(\left(29.4\frac{m}{s}\right)^2 - 2 \cdot 9.8\frac{m}{s^2} \cdot 4.9m\right)}}{9.8\frac{m}{s^2}} = 55\ m$$

Answer: the horizontal range of object will be 55 m.