Answer on Question #46368, Physics, Mechanics | Kinematics | Dynamics

Question

A particle of mass 0.01 kg is moving along the +ve x axis under influence of the force $F(x) = -k(1/2x^2)$,where $k = 0.01\ Nm^2$. At t=0, it is at x = 1 m and velocity = 0.
Find the time at which it reaches x = 0.25.

Solution

According to Newton's second law the acceleration is produced when a force acts on a mass.

$$a=\frac{F}{m}$$

In our case, we have acceleration as a function of distance

$$a(x)=\frac{-k}{2m}x^2$$

The general definition:

$$a(x)=\frac{dv(x)}{dt}=\frac{dv(x)}{dx}\frac{dx}{dt}=\frac{dv(x)}{dx}v(x)$$

Recombine the differentials:

$$a(x)dx=v(x)dv(x)$$

Integrate both parts:

$$\int_{x_0}^{x} a(x)dx=\int_{v_0}^{v} v(x)dv(x)=\frac{v^2-v_0^2}{2}$$

Solve for v(x):

$$v(x)=\sqrt{v_0^2+2\int_{x_0}^{x} a(x)dx}$$

Thus, in our case when $v_0=0$, $x_0=1$, $x=0.25$

$$v(x)=\sqrt{2\int_{1}^{0.25}\frac{-k}{2m}x^2dx}=\sqrt{\frac{-k}{m}\int_{1}^{0.25}x^2dx}=\sqrt{\frac{-k}{m}\left(\frac{x^3}{3}\Big|_1^{0.25}\right)}$$

So,

$$v(0.25)=\sqrt{\frac{-0.01}{0.01}\left(\frac{0.25^3}{3}-\frac{1}{3}\right)}=\sqrt{0.328125}=0.573\,m/s$$

Now, let's find t(x). We will first find it`s derivative:

$$\frac{dt(x)}{dx} = \frac{1}{v(x)}$$

Again, split the differentials and integrate:

$$\int_{t_0}^{t} dt = \int_{x_0}^{x} \frac{dx}{v(x)}$$

Evaluate the leftmost integral:

$$t = t_0 + \int_{x_0}^{x} \frac{dx}{\sqrt{v_0^2 + 2\int_{x_0}^{x} a(x)\,dx}}$$

Thus,

$$t = \int_{0}^{0.25} \frac{dx}{0.573} = \frac{0.25}{0.573} = 0.436\,s$$

Answer: $t = 0.436\,s$.