## Answer on Question \#46198 - Physics - Electromagnetism

Question: a uniform plane wave of 10 kHz travelling in free space strikes a large block of a material having $\epsilon=9 \epsilon_{0}, \mu=4 \mu_{0}$ and $\sigma=0$ normal to the surface. If the incident magnetic field vector is given by

$$
\boldsymbol{B}=10^{-4} \cdot \cos (\omega t-\beta y) \cdot \mathbf{z} \text { tesla. }
$$

Write the complete expressions for the incident, reflected, and transmitted field vectors.
Solution: let us review some general properties of waves that propagate in a homogenous medium. For a $z$-directed, $x$-polarized uniform plane wave incident on a planar media interface located on the $x-y$ plane, the incident, reflected and transmitted fields may be written as (here they are put into phasor form so the time-evolution factor is omitted)
a) incident wave fields:

$$
\begin{aligned}
\widetilde{\boldsymbol{E}}^{i} & =E_{0} e^{-\gamma_{1} z} \cdot \boldsymbol{x} \\
\widetilde{\boldsymbol{H}}^{i} & =\frac{E_{0}}{\eta_{1}} e^{-\gamma_{1} z} \cdot \boldsymbol{y}
\end{aligned}
$$

b) transmitted wave fields:

$$
\begin{aligned}
\widetilde{\boldsymbol{E}}^{t} & =\tau E_{0} e^{-\gamma_{2} z} \cdot \boldsymbol{x} \\
\widetilde{\boldsymbol{H}}^{t} & =\tau \frac{E_{0}}{\eta_{2}} e^{-\gamma_{2} z} \cdot \boldsymbol{y}
\end{aligned}
$$

c) reflected wave fields:

$$
\begin{gathered}
\widetilde{\boldsymbol{E}}^{r}=\Gamma E_{0} e^{\gamma_{1} z} \cdot \boldsymbol{x} \\
\widetilde{\boldsymbol{H}}^{r}=-\Gamma \frac{E_{0}}{\eta_{1}} e^{\gamma_{1} z} \cdot \boldsymbol{y}
\end{gathered}
$$

where $\Gamma$ is the reflection coefficient, $\tau$ - transmission coefficient, $\eta=\sqrt{\frac{\mu}{\epsilon}}$.

a) Let us find first the incident wave fields. Vector $\boldsymbol{H}^{\boldsymbol{i}}$ is equal

$$
\boldsymbol{H}^{\boldsymbol{i}}=\frac{\boldsymbol{B}^{\boldsymbol{i}}}{\mu_{0}}=\frac{10^{-4}}{\mu_{0}} \cdot \cos (\omega t-\beta y) \cdot \boldsymbol{z} \cong 79.56 \cdot \cos (\omega t-\beta y) \cdot \boldsymbol{z} \frac{A}{m} .
$$

$\operatorname{Vector} \boldsymbol{E}^{\boldsymbol{i}}$ has the magnitude $E_{0}=H_{0} \sqrt{\frac{\mu_{0}}{\epsilon_{0}}}=\frac{10^{-4}}{\sqrt{\mu_{0} \epsilon_{0}}}=3 \cdot 10^{4} \frac{\mathrm{~V}}{\mathrm{~m}}$ and has the direction $-\boldsymbol{x}$. Therefore

$$
\boldsymbol{E}^{\boldsymbol{i}}=\frac{10^{-4}}{\sqrt{\mu_{0} \epsilon_{0}}} \cdot \cos (\omega t-\beta y) \cdot(-\boldsymbol{x}) \cong 3 \cdot 10^{4} \cdot \cos (\omega t-\beta y) \cdot(-\boldsymbol{x}) \frac{V}{m}
$$

b) Let us now calculate the reflected field waves. We know that the frequency of reflected and transmitted waves are the same as incidental, but the speed of propagation and the wavelength are changed. We should also calculate the coefficients $\Gamma, \tau$ :

$$
\begin{gathered}
\Gamma=\frac{\eta_{2}-\eta_{1}}{\eta_{2}+\eta_{1}}=\frac{\sqrt{\frac{\mu}{\epsilon}}-\sqrt{\frac{\mu_{0}}{\epsilon_{0}}}}{\sqrt{\frac{\mu}{\epsilon}}+\sqrt{\frac{\mu_{0}}{\epsilon_{0}}}}=\frac{\frac{2}{3} \eta_{1}-\eta_{1}}{\frac{2}{3} \eta_{1}+\eta_{1}}=-0.2 \\
\tau=1+\Gamma=0.8
\end{gathered}
$$

Using the general properties above, we conclude that ( $\beta^{r}=-\beta$ because wave propagates in opposite direction)

$$
\begin{aligned}
\boldsymbol{H}^{r} & =\frac{2 \cdot 10^{-5}}{\mu_{0}} \cdot \cos (\omega t+\beta y) \cdot \mathbf{z} \cong 15.91 \cdot \cos (\omega t+\beta y) \cdot \boldsymbol{z} \frac{A}{m} \\
\boldsymbol{E}^{\boldsymbol{r}} & =\frac{2 \cdot 10^{-5}}{\sqrt{\mu_{0} \epsilon_{0}}} \cdot \cos (\omega t+\beta y) \cdot \boldsymbol{x} \cong 6 \cdot 10^{3} \cdot \cos (\omega t+\beta y) \cdot \boldsymbol{x} \frac{V}{\mathrm{~m}} .
\end{aligned}
$$

c) Finally, let us find the transmitted wave. First of all, new wave number of the wave in the medium is

$$
\beta^{t}=\frac{2 \pi}{\lambda^{t}}=\frac{\omega}{c}=\omega \sqrt{36 \mu_{0} \epsilon_{0}}=6 \beta .
$$

Therefore (using the general case) we conclude

$$
\begin{gathered}
\boldsymbol{E}^{\boldsymbol{t}}=\frac{0.8 \cdot 10^{-4}}{\sqrt{\mu_{0} \epsilon_{0}}} \cdot \cos (\omega t-6 \beta y) \cdot(-\boldsymbol{x}) \cong 3 \cdot 10^{4} \cdot \cos (\omega t-6 \beta y) \cdot(-\boldsymbol{x}) \frac{\mathrm{V}}{\mathrm{~m}} . \\
\boldsymbol{H}^{\boldsymbol{t}}=\frac{2.4 \cdot 10^{-5}}{\mu_{0}} \cdot \cos (\omega t-6 \beta y) \cdot \boldsymbol{z} \cong 19.09 \cdot \cos (\omega t-6 \beta y) \cdot \boldsymbol{z} \frac{A}{\mathrm{~m}} .
\end{gathered}
$$

## Answer:

a) incident waves:

$$
\begin{gathered}
\boldsymbol{E}^{\boldsymbol{i}}=\frac{10^{-4}}{\sqrt{\mu_{0} \epsilon_{0}}} \cdot \cos (\omega t-\beta y) \cdot(-\boldsymbol{x}) \cong 3 \cdot 10^{4} \cdot \cos (\omega t-\beta y) \cdot(-\boldsymbol{x}) \frac{\mathrm{V}}{\mathrm{~m}} . \\
\boldsymbol{H}^{\boldsymbol{i}}=\frac{10^{-4}}{\mu_{0}} \cdot \cos (\omega t-\beta y) \cdot \boldsymbol{z} \cong 79.56 \cdot \cos (\omega t-\beta y) \cdot \boldsymbol{z} \frac{A}{\mathrm{~m}} .
\end{gathered}
$$

b) reflected waves:

$$
\begin{gathered}
\boldsymbol{H}^{r}=\frac{2 \cdot 10^{-5}}{\mu_{0}} \cdot \cos (\omega t+\beta y) \cdot \boldsymbol{z} \cong 15.91 \cdot \cos (\omega t+\beta y) \cdot \boldsymbol{z} \frac{A}{m} . \\
\boldsymbol{E}^{r}=\frac{2 \cdot 10^{-5}}{\sqrt{\mu_{0} \epsilon_{0}}} \cdot \cos (\omega t+\beta y) \cdot \boldsymbol{x}=\cong 6 \cdot 10^{3} \cdot \cos (\omega t+\beta y) \cdot \boldsymbol{x} \frac{\mathrm{V}}{\mathrm{~m}} .
\end{gathered}
$$

c) transmitted waves:

$$
\begin{gathered}
\boldsymbol{E}^{t}=\frac{0.8 \cdot 10^{-4}}{\sqrt{\mu_{0} \epsilon_{0}}} \cdot \cos (\omega t-6 \beta y) \cdot(-\boldsymbol{x}) \cong 3 \cdot 10^{4} \cdot \cos (\omega t-6 \beta y) \cdot(-\boldsymbol{x}) \frac{\mathrm{V}}{\mathrm{~m}} . \\
\boldsymbol{H}^{t}=\frac{2.4 \cdot 10^{-5}}{\mu_{0}} \cdot \cos (\omega t-6 \beta y) \cdot \boldsymbol{z} \cong 19.09 \cdot \cos (\omega t-6 \beta y) \cdot \boldsymbol{z} \frac{A}{\mathrm{~m}} .
\end{gathered}
$$

