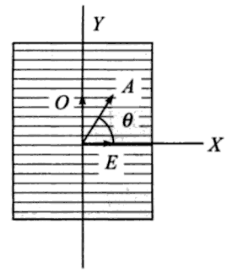
Answer on Question #45659, Physics, Optics

Derive linear circular polarized light from the equation of an ellipse.

Solution:



Suppose that a plane polarized light beam of amplitude A is incident on a uniaxial crystal at an angle θ. Let $A\cos\theta$ and $A\sin\theta$ be the amplitudes of E-ray and O-ray respectively. If δ be the phase difference between the two emergent beams, then their vibrations can be expressed as

For E-ray: $x = A\cos\theta\sin(\omega t+\delta) = a\sin(\omega t+\delta)$ (1)

For O-ray: $y = A\sin\theta\sin\omega t = b\sin\omega t$ (2)

where $a = A\cos\theta$ and $b = A\sin\theta$

From second equation we have:

$$\frac{y}{b} = \sin\omega t$$

Hence $\cos\omega t = \sqrt{1-\sin^2\omega t} = \sqrt{1-\frac{y^2}{b^2}}$

From first eqation we have:

$$x = a\sin(\omega t+\delta) = a(\sin\omega t\cos\delta + \cos\omega t\sin\delta)$$

or,

$$\frac{x}{a} = \sin\omega t\cos\delta + \cos\omega t\sin\delta = \frac{y}{b}\cos\delta + \sqrt{1-\frac{y^2}{b^2}}\sin\delta$$

or,

$$\frac{x}{a} - \frac{y}{b}\cos\delta = \sqrt{1-\frac{y^2}{b^2}}\sin\delta$$

Squaring and rearranging, we get:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\delta = \sin^2\delta$$

This is the **general equation of an ellipse**.

Spesial cases:

1. When δ = 0 sin δ = 0 and cos δ = 1, therefore

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos 0 = \sin^2 0$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0$$

$$\left[\frac{x}{a} - \frac{y}{b}\right]^2 = 0$$

or,

$$y = \frac{b}{a}x$$

This is the **equation of a straight line**. In this case, the emergent light is **plane polarized**.