Answer on Question #45651-Physics-Mechanics-Kinematics-Dynamics

The Father of a 20 kg child stands at the summit of a conical hill as he spins his child around on a 5.0 kg cart with a 2.0m long rope. The sides of the hill are inclined at 20 degrees. He keeps the rope parallel to the ground, and friction is negligible. What rope tension will allow the cart to spin with the 15 rpm?

Solution

Since the unknown is force, you have to look for all the forces acting on the system. For the system, take the child and the cart. The system, therefore, has a total mass of $M = (20 + 5)kg = 25kg$. Now, the forces acting on the system are

(a) weight Mg along the vertical,

(b) tension F_T along the rope, F_T making an angle of 20 deg with the horizontal

(c) normal force F_N perpendicular to the incline, with F_N making an angle of 20 deg with the vertical.

I am using a normal 2 dimensional coordinate system with the x-axis along the horizontal, y-axis along the vertical.

Now, as the system rotates along the side of the hill, it describes a horizontal circle parallel to our x-axis and lying below the summit of the hill. Because of the circular motion, application of Newton's second law gives (assuming the 20 deg is measured relative to the horizontal)

(1) $F_T \cos(20) - F_N \sin(20) = \frac{Mv^2}{R}$

(above equation holds true at the instant the (child + cart) is on the right side of the hill with (+) force toward center of circular motion, (-) force away from center of circular motion).

Along the x-axis, v is the velocity (speed) of circular motion and R is the radius of circular motion lying along the horizontal circle below the summit, so that

(2) $= L\cos(20)\,, L = 2.0\, m.$

Along the vertical (y-axis) we get

(3) $F_T \sin(20) + F_N \cos(20) - Mg = 0$ (no motion along vertical)

which gives

(4) $F_N = \frac{\{Mg - F_T \sin(20)\}}{\cos(20)}$

Substituting eqs.(2) and (4) in eq.(1), we get

(5) $F_T \cos(20) - \{Mg - F_T \sin(20)\} \tan(20) = \frac{Mv^2}{\{Lcos(20)\}}.$

Solving for F_T, we get

(6) $F_T\{\cos(20) + \sin(20)\tan(20)\} = Mg\tan(20) + \frac{Mv^2}{\{Lcos(20)\}}$

Equivalently,

$$F_T \frac{\{[\,cos(20)][\,cos(20)] + [sin(20)][\,sin(20)]\}}{\{\,cos(20)\}} = \frac{\left\{Mgsin(20) + \frac{Mv^2}{L}\right\}}{\{cos(20)\}}.$$

$$\frac{F_T}{cos(20)} = \frac{\left\{Mgsin(20) + \frac{Mv^2}{L}\right\}}{\{\,cos(20)\}}$$

Notice that $cos(20)$ cancels out, leaving us

(7) $F_T = Mg\sin(20) \ + \frac{Mv^2}{L}$

Next, we have to find v before we can get F_T. To do that, we use the given 15 rpm. Now 15 rpm corresponds to 15 rev in a time of 60 s. Since increasing the elapsed time for a given period (time for one full turn) also increases the number of rev, we can use the following ratio and proportion $\left[\frac{1rev}{elapsed\ time\ of\ one\ period\ P}\right] = \left[\frac{15rev}{60\ s}\right]$ we get

$$P \ = \frac{60s}{15} = 4\ s.$$

Moreover, in one rev, the distance traveled is one circumference C of a circle. That means

$$vP \ = \ 2(\pi)R \ = \ 2(\pi)L\,cos(20)$$

which yields

$$v \ = 2(\pi)L\frac{cos(20)}{4} = \ 0.5(\pi)L\,cos(20)$$

$$\frac{v^2}{L} = \ (0.5)^2\pi^2\cos^2(20)\,.$$

Substituting eq.(8) in eq.(7), we finally get

$$F_T \ = \ 25(9.8)(0.342) \ + 25(0.5)^2\pi^2(2.0)(0.93969)^2 = 193\ N.$$

The tension in the rope must be $193\ N$.