Answer on Question #45544, Physics, Quantum Mechanics

If the function ψ_1 and ψ_2 are solution of Schrodinger wave equation for a particle , then prove that $a_1\psi_1 + a_2\psi_2$ is also a solution of same equation, where a_1 and a_2 are arbitrary constants.

Solution

see on next page.

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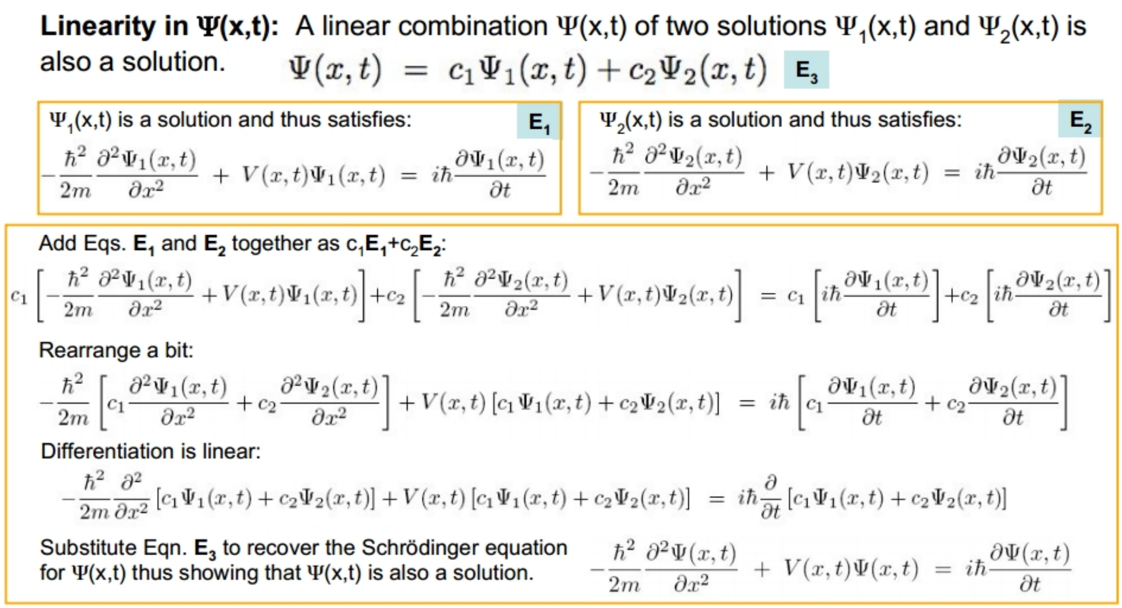
Linearity in Ψ(x,t): A linear combination Ψ(x,t) of two solutions Ψ_1(x,t) and Ψ_2(x,t) is also a solution.
$\Psi(x,t) = c_1\Psi_1(x,t) + c_2\Psi_2(x,t)$ E_3
Ψ_1(x,t) is a solution and thus satisfies: E_1
$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi_1(x,t)}{\partial x^2} + V(x,t)\Psi_1(x,t) = i\hbar\frac{\partial\Psi_1(x,t)}{\partial t}$
Ψ_2(x,t) is a solution and thus satisfies: E_2
$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi_2(x,t)}{\partial x^2} + V(x,t)\Psi_2(x,t) = i\hbar\frac{\partial\Psi_2(x,t)}{\partial t}$
Add Eqs. E_1 and E_2 together as $c_1E_1+c_2E_2$:
$c_1\left[-\frac{\hbar^2}{2m}\frac{\partial^2\Psi_1(x,t)}{\partial x^2} + V(x,t)\Psi_1(x,t)\right]+c_2\left[-\frac{\hbar^2}{2m}\frac{\partial^2\Psi_2(x,t)}{\partial x^2} + V(x,t)\Psi_2(x,t)\right] = c_1\left[i\hbar\frac{\partial\Psi_1(x,t)}{\partial t}\right]+c_2\left[i\hbar\frac{\partial\Psi_2(x,t)}{\partial t}\right]$
Rearrange a bit:
$-\frac{\hbar^2}{2m}\left[c_1\frac{\partial^2\Psi_1(x,t)}{\partial x^2}+c_2\frac{\partial^2\Psi_2(x,t)}{\partial x^2}\right] + V(x,t)\left[c_1\Psi_1(x,t)+c_2\Psi_2(x,t)\right] = i\hbar\left[c_1\frac{\partial\Psi_1(x,t)}{\partial t}+c_2\frac{\partial\Psi_2(x,t)}{\partial t}\right]$
Differentiation is linear:
$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\left[c_1\Psi_1(x,t)+c_2\Psi_2(x,t)\right] + V(x,t)\left[c_1\Psi_1(x,t)+c_2\Psi_2(x,t)\right] = i\hbar\frac{\partial}{\partial t}\left[c_1\Psi_1(x,t)+c_2\Psi_2(x,t)\right]$
Substitute Eqn. E_3 to recover the Schrödinger equation for Ψ(x,t) thus showing that Ψ(x,t) is also a solution.
$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$

Figure 1: a