Answer on Question #45363-Physics-Mechanics-Kinematics-Dynamics

A 75 kg shell is fired with an initial speed of 125 m/s at an angle of 55o above the horizontal. Air resistance is negligible. At its highest point the shell explodes into two fragments, one four times more massive than the other. The heavier fragment lands directly below the point of the explosion. If the explosion exerts forces only in the horizontal direction, how far from the launch point does the lighter fragment land?

Solution

Find the shell's speed (horizontal only) at the highest point before the explosion.

$$v_{xi} = v_0 cos\theta_0 = \left(125\frac{m}{s}\right) cos\ 55° = 72.7\frac{m}{s}.$$

As the explosion exerts horizontal force only, the initial speed of the heavier fragment is zero and the lighter fragment has horizontal component only after the explosion.

Find the speed of the lighter fragment immediately after the explosion.

$$Mv_{xi} = m_1 v_{1xf} + m_2 v_{2xi} = (75\ kg)\left(72.7\frac{m}{s}\right) = \frac{1}{5}(75\ kg)v_{1xf} + \frac{4}{5}(75\ kg)(0)$$

$$(75\ kg)\left(72.7\frac{m}{s}\right) = \frac{1}{5}(75\ kg)v_{1xf} + \frac{4}{5}(75\ kg)(0) \rightarrow v_{1xf} = 358.5\frac{m}{s}.$$

Find the height of the shell when it explodes.

$$v_y^2 = v_{y0}^2 - 2gh,$$

$$0 = \left[\left(125\frac{m}{s}\right) sin55°\right]^2 - \left(9.8\frac{m}{s^2}\right) h \rightarrow h = 535\ m.$$

Find the horizontal distance of the explosion point to the launch point.

$$x_1 = \frac{1}{2}R = \frac{1}{2}\frac{v_0^2 sin\theta_0}{g} = \frac{1}{2}\left(125\frac{m}{s}\right)^2 \frac{sin110°}{9.8\frac{m}{s^2}} = 749\ m.$$

Find the time the lighter fragment in the air.

$$h = \frac{1}{2}gt^2;$$

$$(535\ m) = \frac{1}{2}\left(9.8\frac{m}{s^2}\right)t^2 \rightarrow t = 10.4\ s$$

Find the distance of the lighter fragment it lands from the explosion point.

$$x_2 = v_{1xf}t = \left(358.5\frac{m}{s}\right)(10.4\ s) = 3.73 \cdot 10^3\ m$$

$$x = x_1 + x_2 = 749\ m + 3.73 \cdot 10^3\ m = 4.5 \cdot 10^3\ m.$$

Answer: $4.5 \cdot 10^3\ m$.