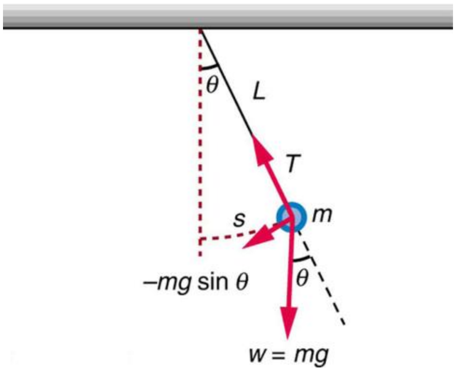
Answer on Question #45307, Physics, Mechanics | Kinematics | Dynamics

A small sphere of mass m suspended by a thread is first taken aside so that the thread forms the right angle with the vertical and then released. Find:

(a) the total acceleration of the sphere and the thread tension as a function of θ, the angle of deflection of the thread from the vertical.

Solution:



Mechanical energy E is the sum of the potential and kinetic energies of an object.

$$E = U + K$$

The total mechanical energy in any isolated system of objects remains constant if the objects interact only through conservative forces:

$$U_f + K_f = U_i + K_i$$

$$mgl = \frac{mv^2}{2} + mgl(1 - \cos\theta)$$

where l is length of the thread.

$$gl = \frac{v^2}{2} + gl(1 - \cos\theta)$$

$$v^2 = 2gl\cos\theta$$

The normal acceleration is

$$a_n = \frac{v^2}{l} = 2g\cos\theta$$

The linear displacement from equilibrium is s, the length of the arc. Also on figure shown are the forces on the ball, which result in a net force of F=mgsinθ toward the equilibrium position—that is, a restoring force.

The tangential acceleration is

$$a_t = \frac{F}{m} = g\sin\theta$$

The total acceleration is

$$a = \sqrt{a_n^2 + a_t^2} = g\sqrt{4\cos^2\theta + \sin^2\theta} = g\sqrt{3\cos^2\theta + \cos^2\theta + \sin^2\theta} = g\sqrt{3\cos^2\theta + 1}$$

The force equation is

$$mg\cos\theta = ma_n$$

Thus,

$$= ma_n + mg\cos\theta = 2mg\cos\theta + mg\cos\theta = 3mg\cos\theta$$

Answer: $a = g\sqrt{3\cos^2\theta + 1}$ **,** $= 3mg\cos\theta.$