Answer on Question #45196, Physics, Mechanics | Kinematics | Dynamics

A player is about to drive into the rim but at a distance of 10ft a defender came his way. So he decided to take a fade away jump shot having a jumping velocity of 11ft/s at 50degrees to the horizontal. He will be releasing the ball at the highest point of his jump; he also leaned back at an angle of 40degrees to the vertical to further himself from the defender. If the player's shooting height is 6ft and takes his shot at 40degrees to the horizontal, how much initial velocity must he exert unto the ball if the hoop is 10ft high from the floor?

Solution:

Projectile motion is a form of motion in which an object or particle (called a projectile) is thrown near the earth's surface, and it moves along a curved path under the action of gravity only.

In projectile motion, the horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other.

The horizontal component of the velocity of the object remains unchanged throughout the motion. The vertical component of the velocity increases linearly, because the acceleration due to gravity is constant (g=32.174 ft/s^2).

Equations related to trajectory motion are given by

$$\text{Horizontal distance,} \quad x = v_{0x}t$$

$$\text{Vertical distance,} \quad y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

where

$$v_{0x} = v_0 \cos\theta$$
$$v_{0y} = v_0 \sin\theta$$

First, let's find out where the "highest point of his jump" is.

The initial speed of the player is $v_0 = 11$ ft/s and the angle is $\theta = 50°$

Thus,

$$v_{0x} = v_0 \cos\theta = 11 \cdot \cos 50° = 7.071 \text{ ft/s}$$
$$v_{0y} = v_0 \sin\theta = 11 \cdot \sin 50° = 8.426 \text{ ft/s}$$

The highest point in the jump will occur when the vertical component of the velocity is zero, so

$$v_{0y}^2 = 2gh$$

and

$$h = \frac{v_{0y}^2}{2g} = \frac{8.426^2}{2 \cdot 32.174} = 1.103 \text{ ft}$$

The time required to reduce the 8.426 ft/s to zero is

$$t = \frac{v_{0y}}{g} = \frac{8.426}{32.174} = 0.262 \text{ s}$$

during this time, the forward motion will be

$$x = v_{0x}t = 7.071 \cdot 0.262 = 1.853 \text{ ft}$$

However, we must now add to his 1.103 ft upward displacement and his 1.853 ft displacement towards the basket, the further effect of his leaning back by 40 degrees.

Let's say the lean-back moves the point from which he shoots, backwards by

$$6 \cdot \sin 40° = 3.857 \text{ ft}$$

and downwards by

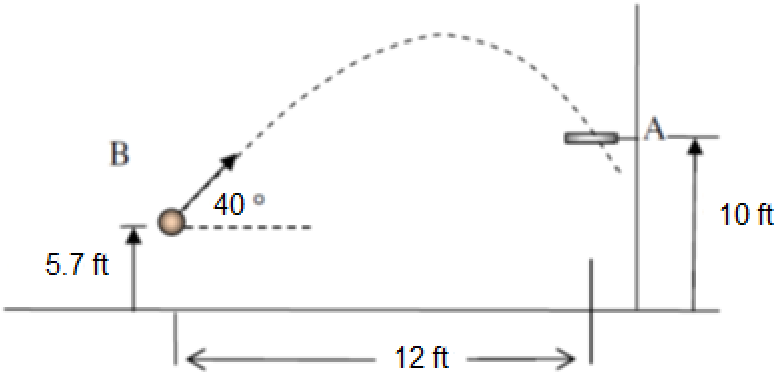
$$6 - 6\cos 40° = 1.404 \text{ ft.}$$

That puts the point from which the ball is shot at a height of

$$y_0 = 6 - 1.404 + 1.103 = 5.699 \text{ ft}$$

and at a horizontal distance from the basket of

$$x_0 = 10 + 3.857 - 1.853 = 12.004 \text{ ft}$$



We use x-y coordinates with origin at the release point.

For x coordinate:

$$[x = x_0 + (v_x)_0 t]$$
$$12 = 0 + (v_0 \cos 40°)t$$

We now find the flight time

$$t = \frac{12}{v_0 \cos 40°}$$

For y coordinate:

$$\left[y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2\right]$$

The initial speed of the ball is

$$\left[v_0 = \frac{x - x_0}{\cos 40°} \sqrt{\frac{g}{2((x - x_0)\tan 40° - (y - y_0))}}\right]$$

$$v_0 = \frac{12}{\cos 40°} \cdot \sqrt{\frac{32.174}{2 \cdot \left(12 \cdot \tan 40° - (10 - 5.7)\right)}} = \frac{12}{\cos 40°} \cdot 1.67 = 26.16 \approx 26.2 \text{ ft/s}$$

Answer. $v_0 = 26.2$ ft/s.