Answer on Question #45163-Physics-Electromagnetism

Deuterons (q=e, m=3.3*10^(-27) kg) with kinetic energy of 12 keV are injected near the center of a cyclotron with B =1.7T and 1.1 m diameter.

(a) Calculate the cyclotron frequency.

(b) Determine the final kinetic energy of a deuteron in electron volts.

Solution

(a) Deuterons with the charge e enter the homogenous magnetic field with the magnetic induction $\vec{B}$ and the velocity $\vec{v}$ (the velocity is perpendicular to the magnetic induction). The size of the magnetic force $\vec{F}_m$ that interacts with the particle is equal to:

$$F_m = evB.$$

The size of the centripetal force F during the uniform movement round the circle is formulated:

$$F = ma = \frac{mv^2}{r}.$$

The magnetic force is the centripetal force in this case and therefore:

$$evB = \frac{mv^2}{r}.$$

The radius of the circle that the particle moves around is:

$$r = \frac{mv}{eB}$$

The time to run around the semicircle (one half of the period) $\frac{T}{2}$ is equal to the circumference of the circle πr divided by the velocity v of the particle:

$$\frac{T}{2} = \frac{\pi r}{v} = \frac{\pi}{v}\frac{mv}{eB} = \frac{\pi m}{eB}.$$

The circulation period T of the particle does not depend on its velocity and its energy. Therefore the time between two flights of the particle past the dees is still equal during the acceleration. The equation for the frequency f is then:

$$f = \frac{1}{T} = \frac{eB}{2\pi m} = \frac{1.6 \cdot 10^{-19}c \cdot 1.7T}{2\pi \cdot 3.3 \cdot 10^{-27}kg} = 13.1 \cdot 10^6 Hz = 13.1\ MHz.$$

(b) The radius R of the cyclotron corresponds to the radius of a circle R_{final} when the particle has the final energy. The size of the cyclotron can be expressed from the relation for the radius of the circle from the last section:

$$R_{final} = \frac{mv}{eB}.$$

If the deuteron has the final energy, the velocity of the particle will be:

$$E_{final} = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{eBR_{final}}{m}\right)^2 = \frac{\left(eBR_{final}\right)^2}{2m} = \frac{\left(1.6 \cdot 10^{-19}c \cdot 1.7T \cdot \frac{1.1}{2}\ m\right)^2}{2 \cdot 3.3 \cdot 10^{-27}kg}$$

$$= 3.4 \cdot 10^{-12}J = 21.2 \cdot 10^6 eV = 21.2\ MeV.$$