Answer on Question #44876 – Physics - Other

a) (i)A harmonic wave on a rope is described by the expression

y(x ,t)=(4.3 mm)sin[2pie/0.82m(x+12ms^-1)t]

What are the wave's wavelength, period, wave number, frequency, and direction of propagation.

(ii) For the wave in 2(a) (i) above, determine the displacement and acceleration of the element of the rope located at x = 0.58m at the instant t = 0.41s.

Solution:
1. Traveling sinusoidal wave is represented mathematically in terms of its velocity v (in the □ direction) and wave number k as:

$$y(x,t) = A\sin(k(x - vt))$$

In our case equation of a wave is:

$$y(x,t) = 4.3\text{mm} \cdot \sin\left[\frac{2\pi}{0.82\text{m}}\left(x + \left(12\frac{\text{m}}{\text{s}}\right)t\right)\right]$$

sign "+" means wave moving to left (opposite axis direction).

Therefore, wave number k equals:

$$k = \frac{2\pi}{0.82\text{m}}$$

Wavelength λ equals:

$$\lambda = \frac{2\pi}{k} = 0.82\ \text{m}$$

Period equals:

$$T = \frac{0.82\ \text{m}}{12\frac{\text{m}}{\text{s}}} = 0.068\ \text{s}$$

Frequency equals:

$$f = \frac{1}{T} = \frac{12\frac{\text{m}}{\text{s}}}{0.82\ \text{m}} = 14.63\frac{1}{\text{s}}$$

2. Displacement at x = 0.58m and t = 0.41s equals:

$$y(x,t) = 4.3\text{mm}\sin\left[\frac{2\pi}{0.82\text{m}}\left(0.52\text{m} + \left(12\frac{\text{m}}{\text{s}}\right)0.41\text{s}\right)\right] = -4.15\text{mm}$$

Acceleration equals:

$$a = \frac{d^2}{dt^2}\left(y(t)\right)$$

$$= -4.3\text{mm}\left(\frac{2\pi \cdot 12\frac{\text{m}}{\text{s}}}{0.82\text{m}}\right)\sin\left[\frac{2\pi}{0.82\text{ m}}\left(x + \left(12\frac{\text{m}}{\text{s}}\right)\right)t\right]\ 0.82\text{m}\ 0.82\text{m s}$$

Acceleration at x = 0.58m and t = 0.41s equals:

$$a = 382\frac{\text{mm}}{\text{s}^2}$$

Answer:

1. Wavelength: $\lambda = 0.82\text{ m}$

Period: $T = 0.068\text{ s}$

Frequency: $f = 14.63\frac{1}{\text{s}}$

2. $y(x,t) = -4.15\text{mm}$, $a = 382\frac{\text{mm}}{\text{s}^2}$