

Answer on Question #44793, Physics, Other

Question:

The displacement of a particle executing SHM is given by

$$x(t) = 6 \sin 5\pi t + 8 \cos 5\pi t$$

where x is in cm and t is in seconds. Calculate the amplitude, time period and initial phase of the SHM. Also, obtain the expression for the velocity of the particle.

Answer:

The displacement of a particle can be express as:

$$x(t) = \frac{10}{10} (6 \sin 5\pi t + 8 \cos 5\pi t) = 10 \left(\frac{3}{5} \sin 5\pi t + \frac{4}{5} \cos 5\pi t \right)$$

$$\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = 1, \text{ thus}$$

$$x(t) = 10(\cos \alpha \sin 5\pi t + \sin \alpha \cos 5\pi t) = 10 \sin(5\pi t + \alpha)$$

$$\text{where } \alpha = \arctan \frac{4}{3}$$

Therefore amplitude equals:

$$A = 10 \text{ cm}$$

and initial phase equals:

$$\alpha = \arctan \frac{4}{3} \cong 53.13^\circ$$

time period equals:

$$T = \frac{2\pi}{5\pi} = 0.4 \text{ s}$$

velocity of the particle equals:

$$v = \frac{dx}{dt} = \frac{d}{dt} (10 \sin(5\pi t + \alpha)) = 50\pi \cos(5\pi t + \alpha)$$