

## Answer on Question # 44772, Physics, Electric Circuits

**Task:**  $V_{\text{steady}} = [1/(C^2R^2\omega^2)+1]\sin\omega t + [CR\omega/(C^2R^2\omega^2)+1]\cos\omega t$

In steady state, show that the amplitude of the output voltage?

explain if this circuit is a high-pass or low-pass filter?

**Solution:**

- If the output voltage is given by

$$v_{\text{out}}(t) = B \sin(\omega t), \text{ let } B=1;$$

- Then, since the output voltage is across a capacitor, we can compute the current flowing through R and C as:

$$i(t) = Cdv_{\text{out}}/dt = C\omega \cos(\omega t)$$

- And then we can compute the voltage across the resistor, R, as:

$$v_R(t) = Ri(t) = RC \omega \cos(\omega t)$$

Now, we can apply KVL to get the input voltage.

- The input voltage is given by,

$$v_{\text{in}}(t) = v_R(t) + v_{\text{out}}(t)$$

- Or:

$$v_{\text{in}}(t) = (RC \omega \cos(\omega t) + \sin(\omega t))$$

It may not be obvious, but we can take advantage of a trigonometric identity,

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

if only we can make the things that multiply the sines and cosines in the second bullet above look like other sines and cosines.

- We know:

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

- And, we know:

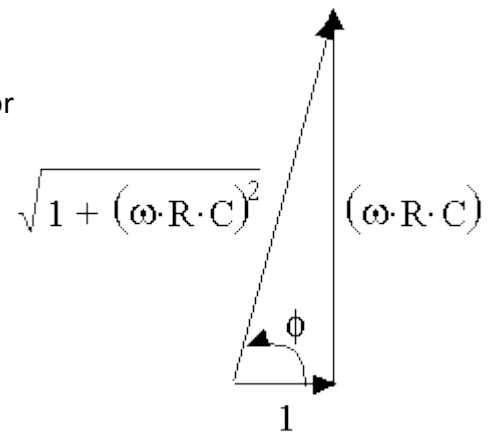
$$v_{\text{in}}(t) = (RC\omega \cos(\omega t) + \sin(\omega t))$$

- And the second expression can be put into the form of the first

We need to refer to a little geometrical construction - at the right. "Clearly" we have the relationships indicated below for  $\cos(\phi)$  and  $\sin(\phi)$

$$\sin(\phi) = \frac{(\omega \cdot R \cdot C)}{\sqrt{1 + (\omega \cdot R \cdot C)^2}}$$

$$\cos(\phi) = \frac{1}{\sqrt{1 + (\omega \cdot R \cdot C)^2}}$$



So, now we can write:  $V_{\text{steady}} = [1/(C^2R^2\omega^2)+1]\sin\omega t + [CR\omega/(C^2R^2\omega^2)+1]\cos\omega t$ .

In a low-pass filter, if the output is taken across the capacitor, and the low frequency components appear across the capacitor, it's a low-pass filter.