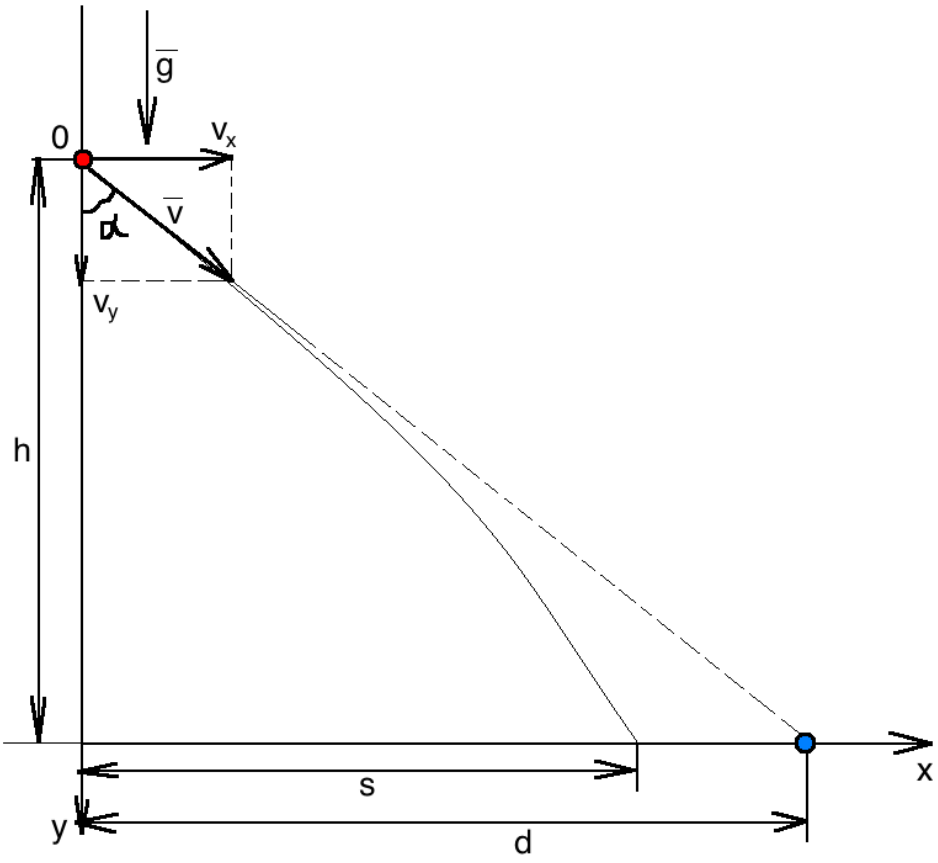
Answer on Question #44761 – Physics - Mechanics | Kinematics | Dynamics

a boy standing on the top of a tower of height 54 ft. throws a packet with a speed of 20 ft/s directly aiming towards his friend standing on the ground at a distance of 72 ft from the foot of the tower. the packet falls short of the person on the ground by x*16/3 ft. the value of x is

Solution:



$h = 54$ ft. $-$height of the tower;

$V = 20\dfrac{\text{ft}}{\text{s}} -$ initial velocity of the packet;

$d = 72$ ft. $-$distance from the tower to friend;

$s = X \cdot \dfrac{16}{3} -$ distance from packet to friend;

$\alpha -$ angle between vertical and direction of the motion;

Components of the velocity along X-axis and Y-axis:

$$V_x = V\cos\alpha;\ V_y = V\sin\alpha;$$

From the right triangle:

$$\sin\ \alpha = \frac{d}{AB} = \frac{d}{\sqrt{d^2 + h^2}} = \frac{72\text{ ft}}{\sqrt{(72\text{ ft})^2 + (54\text{ ft.})^2}} = \frac{4}{5}$$

$$\cos\alpha = \frac{h}{AB} = \frac{h}{\sqrt{d^2 + h^2}} = \frac{54\text{ ft}}{\sqrt{(72\text{ ft})^2 + (54\text{ ft.})^2}} = \frac{3}{5}$$

$$\tan\alpha = \frac{d}{h} = \frac{72\text{ ft.}}{54\text{ ft.}} = \frac{4}{3}$$

Equation of motion of the particle along X-axis:

$$x: s = V_x t = Vt\sin\alpha$$

$$t = \frac{s}{V\sin\alpha} \quad (1)$$

Equation of motion of the particle along Y-axis ($g = 32\frac{\text{ft}}{s^2}$):

$$y: h = Vt\cos\alpha + \frac{gt^2}{2} \quad (2)$$

$$(1)\text{in}(2):$$

$$h = V\frac{s}{V\sin\alpha}\cos\alpha + \frac{g(\frac{s}{V\sin\alpha})^2}{2}$$

$$h = s\tan\alpha + \frac{gs^2}{2V^2\sin^2\alpha}$$

$$54 = \frac{4}{3}\cdot\frac{16}{3}X + \frac{32\frac{ft}{s^2}\cdot\left(\frac{16}{3}X\right)^2}{2\cdot\left(20\frac{\text{ft}}{s}\right)^2\cdot\left(\frac{4}{5}\right)^2}$$

$$54 = \frac{16}{9}X(X+4)$$

$$\frac{16}{9}X^2 + \frac{64}{9}X - 54 = 0$$

Solutions of the quadratic equation:

$$X_1 = \frac{1}{4}\left(-8 - 5\sqrt{22}\right) \approx -7.86$$

$$X_2 = \frac{1}{4}\left(5\sqrt{22} - 8\right) \approx 3.86$$

We need only positive root, hence $X = 3.86$.

Answer: The value X is 3.86