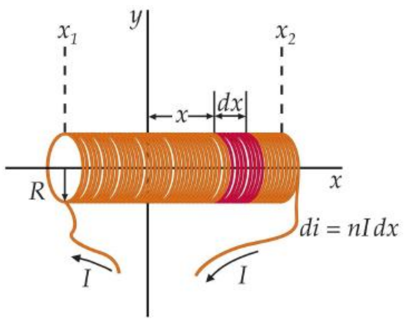
Answer on Question #44692-Physics-Electromagnetism

A long solenoid of turn density 'n' carries a current 'I'. The magnetic field at a point which is at a axial distance R from one end of solenoid? Radius is also R

Solution



Number of turns per unit length: $n = \frac{N}{L}$.

Current circulating in ring of width dx': $nIdx'$.

Magnetic field on axis of ring:

$$dB_x = \frac{\mu_0(nIdx')}{2}\frac{R^2}{[(x - x')^2 + R^2]^{\frac{3}{2}}}.$$

Magnetic field on axis of solenoid:

$$B_x = \frac{\mu_0(nI)R^2}{2}\int_{x_1}^{x_2}\frac{dx'}{[(x - x')^2 + R^2]^{\frac{3}{2}}} = \frac{\mu_0(nI)}{2}\left[\frac{x - x_1}{\sqrt{(x - x_1)^2 + R^2}} - \frac{x - x_2}{\sqrt{(x - x_2)^2 + R^2}}\right].$$

$$x - x_1 = L + R, x - x_2 = R.$$

As solenoid is long ($L \gg R$)

$$\frac{L + R}{\sqrt{(L + R)^2 + R^2}} \to \frac{L + R}{\sqrt{(L + R)^2}} = 1.$$

That's why

$$B = \frac{\mu_0(nI)}{2}\left[1 - \frac{R}{\sqrt{(R)^2 + R^2}}\right] = \frac{\mu_0(nI)}{2}\left[1 - \frac{1}{\sqrt{2}}\right].$$

Answer: $\boldsymbol{\frac{\mu_0(nI)}{2}\left[1 - \frac{1}{\sqrt{2}}\right]}$.