Answer on Question #44668, Physics, Mechanics | Kinematics | Dynamics

1.The velocity of a particle moving on the x -axis is given by v = x^2 + x, where 'x' is in m and 'v' is in m/s
What is the position (in meters) when it's acceleration is 30 m/(s^2)?

2. A boy standing on the top of a tower of height 54 ft. throws a packet with a speed of 20 ft./s directly
aiming for his friend who is standing 72 ft. away from the foot of the tower. The packet falls short of the friend on the ground by X*16/3. The value X is _______ .

Solution:

#1

$a_1 = 30\frac{m}{s} - \text{final acceleration of the particle;}$

$x_1 - \text{position when it'sacceleration } a_1;$

The velocity of a particle moving on the x –axis:

$$v = x^2 + x \quad (1)$$

The acceleration of the particle is $\left(v = \frac{dx}{dt}\right)$

$$a = v'(t) = \frac{d(x^2 + x)}{dt} = 2x \cdot \frac{dx}{dt} + \frac{dx}{dt} = 2x \cdot v + v \quad (2)$$

$$(1)\text{in}(2):$$

$$a = 2x \cdot (x^2 + x) + x^2 + x = 2x^3 + 2x^2 + x^2 + x = 2x^3 + 3x^2 + x$$

For the acceleration $a_1 = 30\frac{m}{s}$:

$$a_1 = 2{x_1}^3 + 3{x_1}^2 + x_1$$

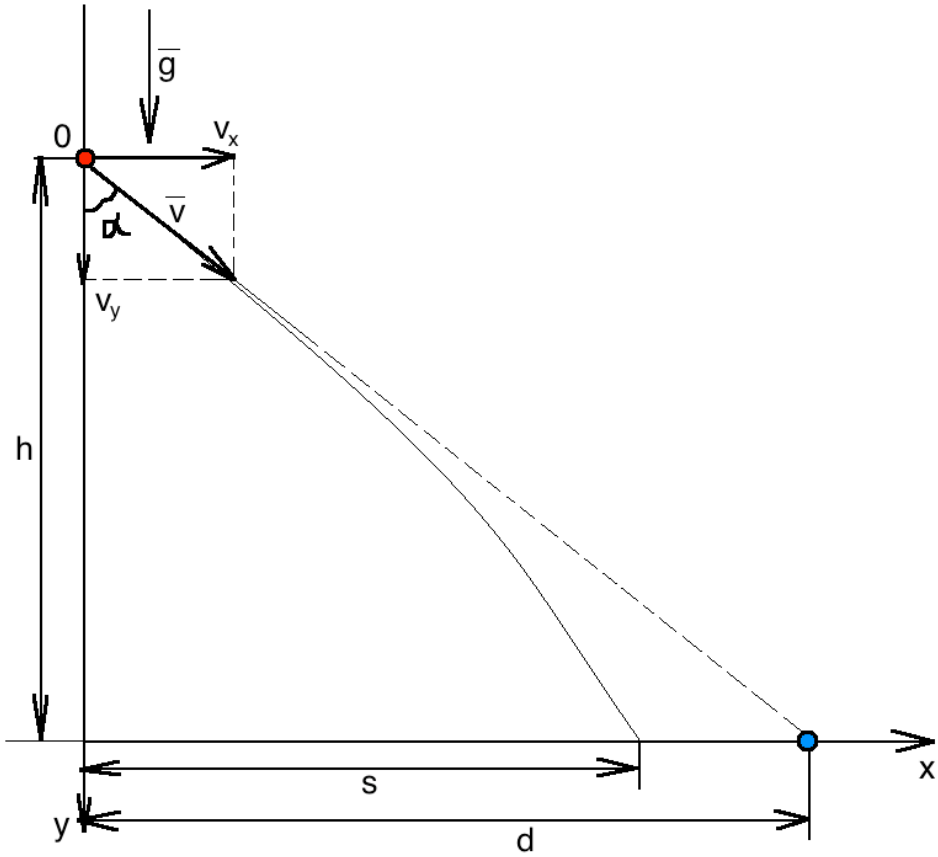
$$2{x_1}^3 + 3{x_1}^2 + x_1 - 30 = 0$$

Real solution of the equation:

$$x_1 = 2$$

Answer: the position (in meters) when it's acceleration is $30\frac{m}{s}$ is 2m.

#2



$h = 54$ ft. —height of the tower;

$V = 20\frac{ft}{s}$ — initial velocity of the packet;

$d = 72$ ft. —distance from the tower to friend;

$s = X \cdot \frac{16}{3}$ — distance from packet to friend;

α — angle between vertical and direction of the motion;

Components of the velocity along X-axis and Y-axis:

$$V_x = V\cos\alpha;\ V_y = V\sin\alpha;$$

From the right triangle:

$$\sin\ \alpha = \frac{d}{AB} = \frac{d}{\sqrt{d^2 + h^2}} = \frac{72\text{ ft}}{\sqrt{(72\text{ ft})^2 + (54\text{ ft.})^2}} = \frac{4}{5}$$

$$\cos\alpha = \frac{h}{AB} = \frac{h}{\sqrt{d^2 + h^2}} = \frac{54\text{ ft}}{\sqrt{(72\text{ ft})^2 + (54\text{ ft.})^2}} = \frac{3}{5}$$

$$\tan\alpha = \frac{d}{h} = \frac{72\text{ ft.}}{54\text{ ft.}} = \frac{4}{3}$$

Equation of motion of the particle along X-axis:

$$x: s = V_x\, t = Vt\sin\alpha$$

$$t = \frac{s}{V \sin\alpha} \quad (1)$$

Equation of motion of the particle along Y-axis ($g = 32\frac{ft}{s^2}$):

$$y: h = Vt\cos\alpha + \frac{gt^2}{2} \quad (2)$$

$$(1) in (2):$$

$$h = V\frac{s}{V\sin\alpha}\cos\alpha + \frac{g(\frac{s}{V\sin\alpha})^2}{2}$$

$$h = s\tan\alpha + \frac{gs^2}{2V^2\sin^2\alpha}$$

$$54 = \frac{4}{3}\cdot\frac{16}{3}X + \frac{32\frac{ft}{s^2}\cdot\left(\frac{16}{3}X\right)^2}{2\cdot\left(20\frac{ft}{s}\right)^2\cdot\left(\frac{4}{5}\right)^2}$$

$$54 = \frac{16}{9}X(X+4)$$

$$\frac{16}{9}X^2 + \frac{64}{9}X - 54 = 0$$

Solutions of the quadratic equation:

$$X_1 = \frac{1}{4}\left(-8 - 5\sqrt{22}\right) \approx -7.86$$

$$X_2 = \frac{1}{4}\left(5\sqrt{22} - 8\right) \approx 3.86$$

We need only positive root, hence $X = 3.86$.

Answer: The value X is 3.86