Answer on Question #44556-Physics-Other

Establish the differential equation for a system executing simple harmonic motion (SHM). Show that, for SHM, the velocity and acceleration of the oscillating object is proportional to ω_0 and ω_0^2, respectively, where ω_0 is the natural angular frequency of the object.

Solution

Simple harmonic motion is typified by the motion of a mass on a spring when it is subject to the linear elastic restoring force given by Hooke's Law.

Now since $F = -kx$ is the restoring force and from Newton's law of motion force is give as $F = ma$,

where m is the mass of the particle moving with acceleration a. Thus acceleration of the particle is

$$a = \frac{F}{m} = \frac{-kx}{m}$$

but we know that acceleration $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$.

Thus,

$$\frac{d^2x}{dt^2} = \frac{-kx}{m}.$$

This differential equation is known as the simple harmonic equation.

The solution is

$$x = A\cos(\omega_0 t + \varphi)$$

where A, ω_0 and φ are all constants.

We know that velocity of a particle is given by

$$v = \frac{dx}{dt}.$$

Now differentiating the displacement of particle x with respect to t

$$v = \frac{dx}{dt} = -A\omega_0 \sin(\omega_0 t + \varphi).$$

From trigonometry we know that

$$\sin^2 x + \cos^2 x = 1.$$

Thus,

$$A^2 \sin^2(\omega_0 t + \varphi) = A^2 - A^2\cos^2(\omega_0 t + \varphi) = A^2 - x^2.$$

Or

$$\sin(\omega_0 t + \varphi) = \sqrt{1 - \frac{x^2}{A^2}}$$

Putting this in equation for velocity we get,

$$v = -A\omega_0 \sqrt{1 - \frac{x^2}{A^2}}$$

So it is proportional to ω_0.
Again we know that acceleration of a particle is given by

$$a = \frac{dv}{dt} = \frac{d}{dt}(-A\omega_0 \sin(\omega_0 t + \varphi)) = -A\omega_0^2 \cos(\omega_0 t + \varphi) = -\omega_0^2 x.$$

So it is proportional to ω_0^2.