Answer on Question #44333, Physics, Mechanics | Kinematics | Dynamics

A body dropped from the top of the tower clears 9/25th of the total height in its last second of flight. The height of the tower is ? (g=9.8 m/s^2) and the answer is 122.5.

Solution:

An object in free fall experiences an acceleration g of 9.8 m/s^2. (The - sign indicates a downward acceleration.)

The kinetic equation is

$$y = y_0 + v_o t + \frac{1}{2}at^2$$

where

$y_0 = h = ?$ is initial position

$v_0 = 0\ m/s$ is initial speed

$a = g = 9.8\ m/s^2$ is acceleration

At time t the position of a ball is $y = 0$,

and at time $t - 1\ s$ the position of a ball is $y = \left(1 - \frac{9}{25}\right)h = \frac{16}{25}h$

Thus,

$$\frac{9}{25}h = v_{o1}t + \frac{1}{2}gt^2,$$

where t = 1 second. We need to find v_{o1}, the initial speed as the body enters that last 9/25 of h the height of the tower.

And,

$$v_{o1} = \sqrt{2g\frac{16}{25}h}$$

assuming the drop means no initial speed at the top. Note 16/25h is the height the body dropped up to the last second.

So,

$$\frac{9}{25}h - \sqrt{2g\frac{16}{25}h} - \frac{1}{2}g = 0$$

$$\frac{9}{25}h - \frac{4}{5}\sqrt{2gh} - 4.9 = 0$$

We define h = x^2 so we rewrite to quadratic equation:

$$\frac{9}{25}x^2 - \frac{4}{5}\sqrt{2 \cdot 9.8}x - 4.9 = 0$$

Thus,

$$0.36x^2 - 3.54175x - 4.9 = 0$$

which we solve for x.

$$x_{1,2} = \frac{3.54175 \pm \sqrt{3.54175^2 + 4 \cdot 0.36 \cdot 4.9}}{2 \cdot 0.36}$$

$$x_1 = 11.068$$

$$x_2 = -1.22977$$

and $h = x^2 = 11.068^2 = 122.5$ m.

Answer: $h = 122.5$ m.