

Answer on Question #44197-Physics-Mechanics-Kinematics-Dynamics

For hydrogen iodide, $1\text{H}^{127}\text{I}$, force constant is 314 N m^{-1} . Calculate the fundamental frequency in cm^{-1} unit for

i) $1\text{H}^{127}\text{I}$

ii) $2\text{H}^{127}\text{I}$

Hint: Assume that force constant does not change with isotopic substitution.

Solution

In the harmonic approximation, the stationary vibrational energy levels for a diatomic molecule A–B are given by

$$E_v = (v + \frac{1}{2}) \hbar \omega = (v + \frac{1}{2}) \hbar \sqrt{\frac{k}{\mu}}, \quad v = 0, 1, 2, \dots$$

where k is the “force constant” for the chemical bond between the atoms A and B, μ is the reduced mass, $\mu = m_A m_B / (m_A + m_B)$.

i)

$$\omega_1 = \frac{1}{2\pi c} \sqrt{\frac{k}{\mu_1}} = \frac{1}{2\pi \cdot 3 \cdot 10^8} \sqrt{\frac{314}{\frac{1.67 \cdot 10^{-27} \cdot 212 \cdot 10^{-27}}{1.67 \cdot 10^{-27} + 212 \cdot 10^{-27}}}} = 0.231 \cdot 10^6 \text{ m}^{-1} = 2310 \text{ cm}^{-1}.$$

ii)

$$\omega_2 = \frac{1}{2\pi c} \sqrt{\frac{k}{\mu_2}} = \frac{1}{2\pi \cdot 3 \cdot 10^8} \sqrt{\frac{314}{\frac{3.34 \cdot 10^{-27} \cdot 212 \cdot 10^{-27}}{3.34 \cdot 10^{-27} + 212 \cdot 10^{-27}}}} = 0.164 \cdot 10^6 \text{ m}^{-1} = 1640 \text{ cm}^{-1}.$$